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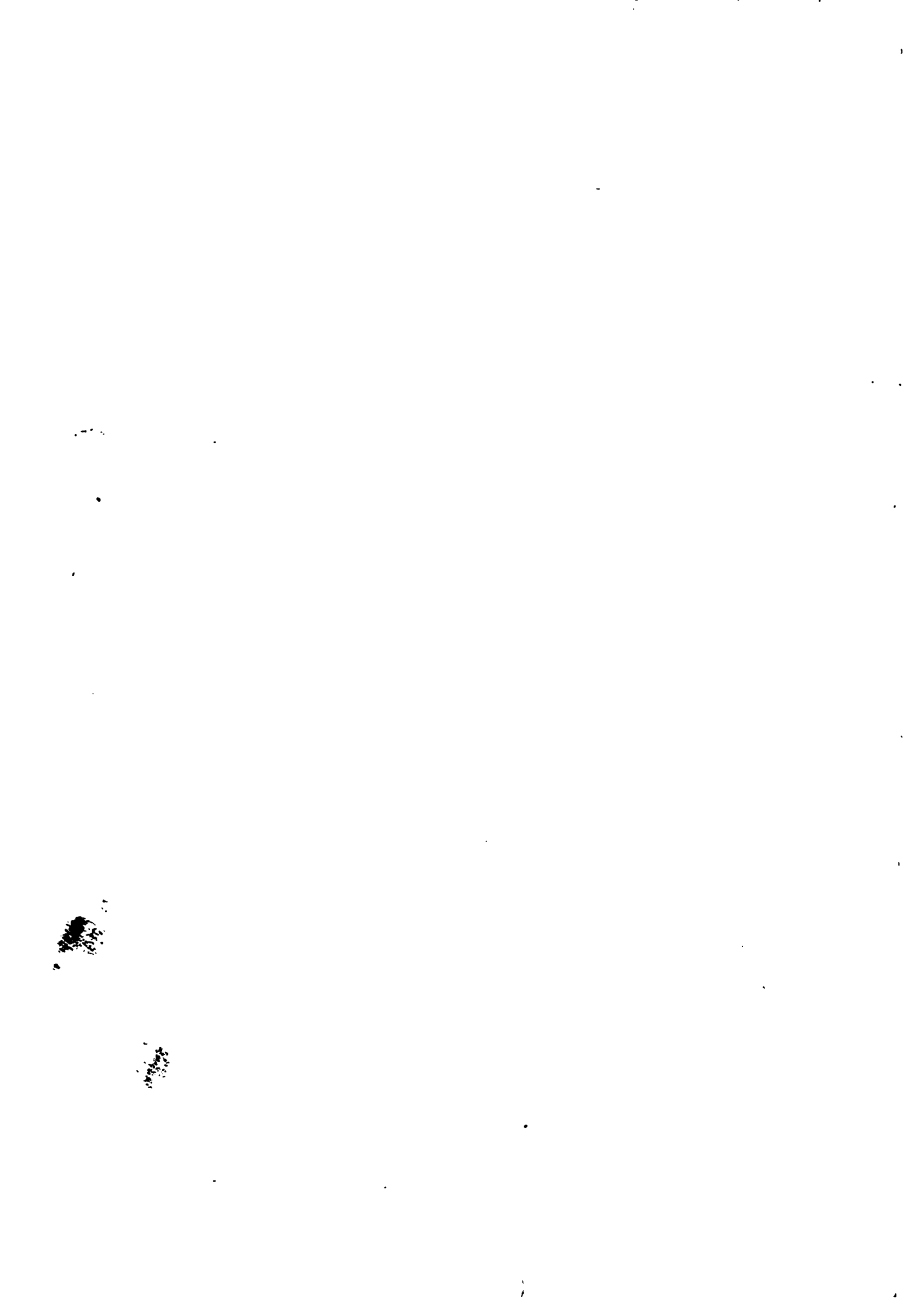
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INTERMEDIATE ARITHMETIC

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PREFACE.

THIS book has been prepared to meet the demands of schools where the first work in number is taught orally. It begins at the point where students enter upon the study of what is commonly called written arithmetic.

It contains a simple but logical presentation of the topics included within its scope. An abundance of written exercises has been supplied to secure expertness in computation, and a great number of oral examples to inspire the pupil with confidence in his ability to reason correctly.

The book is comprehensive enough to meet the needs of those who, while they cannot remain in school long enough to become thoroughly versed in the science of arithmetic, yet desire to gain a knowledge of the principles and processes relating to practical life.

The "Intermediate Arithmetic" serves a double purpose. It is a comprehensive elementary book, and a thorough preparation for the more rigid work of the author's "Standard Arithmetic."

WILLIAM J. MILNE.

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INTERMEDIATE ARITHMETIC.

NOTATION AND NUMERATION.

1. A single thing is called a **Unit**.

Thus, one, a pencil, a tree, a quart, are units.
Units are also sometimes called *ones*.

2. A unit or a collection of units is called a **Number**.

A number answers the question "How many?"

A number may be expressed by *words*, and represented by *figures* and *letters*.

3. The method of representing numbers by means of figures or letters is called **Notation**.

4. The method of reading numbers is called **Numeration**.

THE ARABIC NOTATION.

5. The method of representing numbers by means of figures is called the **Arabic Notation**.

It is called the Arabic Notation because the Arabs first introduced it into Europe.

6. In this notation ten figures are employed to represent numbers, viz.:

FIGURES: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

NAMES: Naught, One, Two, Three, Four, Five, Six, Seven, Eight, Nine.

Each of these, except naught, is called a *significant figure*.

Naught is also called *zero* and *cipher*.

7. The number that is *one* more than *nine* is called **Ten**.

1 ten is called *ten*, and is written, 10.

2 tens are called *twenty*, and are written, 20.

3 tens are called *thirty*, and are written, 30.

4 tens are called *forty*, and are written, 40.

5 tens are called *fifty*, and are written, 50.

6 tens are called *sixty*, and are written, 60.

7 tens are called *seventy*, and are written, 70.

8 tens are called *eighty*, and are written, 80.

9 tens are called *ninety*, and are written, 90.

The suffix *ty* means *ten*. Thus, *forty* means *four tens*; etc.

8. It will be observed that the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, which standing alone represent units, are made to represent tens by writing 0, meaning zero units, after them.

Thus, 50 means five tens and zero units, or five tens.

9. Instead of 0 any of the figures may be written in the right-hand place to represent units. Hence,

When two figures are written side by side, the one at the right represents units and the one at the left tens.

EXERCISES.

10. 1. In 25, what does the figure 2 represent? What does 5 represent? Read the number, beginning at the left.

2. In 72, what does 7 represent? What does 2 represent? Read the number, beginning at the left.

3. In 38, what does 3 represent? What does 8 represent? Read the number, beginning at the left.

4. Tell what each figure represents in the following, and read each number, beginning at the left:

21	62	77	39	55	46
43	28	54	94	88	79
36	23	34	63	57	81
40	30	50	60	20	90

11. Numbers between 1 ten and 2 tens are named thus:

- 1 ten and 1 unit, or 11, eleven.
- 1 ten and 2 units, or 12, twelve.
- 1 ten and 3 units, or 13, thirteen.
- 1 ten and 4 units, or 14, fourteen.
- 1 ten and 5 units, or 15, fifteen.
- 1 ten and 6 units, or 16, sixteen.
- 1 ten and 7 units, or 17, seventeen.
- 1 ten and 8 units, or 18, eighteen.
- 1 ten and 9 units, or 19, nineteen.

The words thirteen, fourteen, fifteen, etc., mean *three* and *ten*, *four* and *ten*, *five* and *ten*, etc.

12. In reading numbers, the word *and* between the tens and the units and the word *units* are omitted.

Thus, 35 is read thirty-five, instead of thirty and five.

EXERCISES.

13. 1. Read the following:

23	41	37	85	96
44	38	92	47	55
16	18	71	17	26
19	52	84	49	99
70	12	48	15	11

2. Write the following, employing figures:

Three tens and one unit.	Thirty-three.	Seventy.
Five tens and five units.	Seventy-one.	Sixty-two.
Eight tens and six units.	Twenty-two.	Eighty-four.
Two tens and seven units.	Sixty-five.	Eighteen.
Six tens and eight units.	Forty-nine.	Thirty-seven.
Nine tens.	Ninety-nine.	Twelve.
Four tens and two units.	Fifty-three.	Seventy-six.

3. Write all numbers between twenty and fifty.

4. Write all numbers between 80 and 99.

5. Express the following numbers in words:

15, 19, 65, 33, 14, 81.

6. Make the following numbers larger by ten, and write them: 4, 8, 7, 2, 9, 1.

7. What number comes next before 10? Before 20? Before 30? Before 80? Before 90?

8. What is the largest number that can be expressed by one figure?

14. What is the smallest number that can be expressed by two figures? What is the largest number that can be expressed by two figures?

99 If we add 1 to 99, we have 9 units and 1, making
1 ten units, or one ten. Adding the one ten
 100 and the nine tens, we have ten tens.

15. Ten tens are called **One Hundred**.

In writing hundreds, the *figure* representing *hundreds* is placed in the *third* place from the right.

Thus, one hundred is written 100.

16. Write with figures:

Two hundred.	Three hundred.	Nine hundred.
Seven hundred.	Four hundred.	Six hundred.
Five hundred.	Eight hundred.	One hundred.

17. *When numbers are expressed by three figures, the figure at the right represents ones, or units, the second figure tens, and the third figure hundreds.*

18. In reading numbers expressed by three figures, the tens and units are read after the hundreds without the word *and*.

251 is read two hundred fifty-one, not two hundred *and* fifty-one.

EXERCISES.

19. 1. Tell what each figure in the following represents, and read each number :

125	275	387	111
205	625	991	400
632	287	444	784
328	346	115	800
407	101	167	776
630	222	822	399
532	461	349	401

2. Write the following numbers, placing units under units, tens under tens, etc.:

Three hundred forty-six.	Sixteen.
Five hundred eighteen.	Six hundred eighty-five.
Ninety-two.	Forty-seven.
One hundred seventy-five.	Seven hundred sixteen.
Two hundred eighty.	Two hundred two.
Seven hundred eleven.	Four hundred eleven.
Four hundred forty-four.	Eight hundred.
Three hundred five.	Five hundred twenty-nine.
Eight hundred seventeen.	Nine hundred seventy-three.
Six hundred nine.	Seven hundred sixty-four.
Three hundred twenty-five.	Two hundred sixty.
Five hundred one.	Three hundred sixty-five.
Four hundred fifty-seven.	Eight hundred six.
Nine hundred nineteen.	Six hundred sixty-six.
Nine hundred ninety-nine.	One hundred one.

3. Write in columns of ten numbers each all the numbers from one hundred to one hundred forty-nine.

4. From two hundred fifty-one to three hundred.

5. What number is next greater than 549, 328, 191, 400, 589, 240, 379, 599, 109, 765 ?

6. Make the following numbers larger by one hundred, and write them: 17, 89, 30, 4, 52, 305.

7. What number comes next before 100, 700, 201, 870, 310?

8. How many hundreds are there in each of the following numbers? How many tens? How many units?

468	453	754	902	999	837	964	815
125	602	882	176	731	395	618	274

20. What is the smallest number you can express by three figures? The largest?

999 If we add 1 to 999, we have nine units and 1,
 1 making ten units or one ten. Adding the one ten and
 1000 the nine tens, we have ten tens or one hundred. Adding
 the one hundred to the nine hundreds, we have
 ten hundreds, which we call by the new name **one thousand**.

7000 is read, seven thousand. 1720 is read, one thousand seven hundred twenty.

EXERCISES

Read the following:

1.	2.	3.	4.	5.	6.	7.
2000	3000	5000	7000	4000	8000	9000
3791	4809	7002	9100	4700	2083	1005
4263	5074	2380	1234	5678	7985	9090

8. What place, counting from the right, do thousands occupy?

9. What is the smallest whole number that can be expressed by four figures? What is the greatest?

10. How many units, tens, and hundreds are expressed by each of the following numbers?

Write and read:

11.	12.	13.	14.	15.	16.	17.
6709	7891	7000	7777	1098	5055	9100
7253	8005	7700	4028	7005	7643	1001
1389	6666	7770	2003	4809	5010	1110

18. Read the numbers in 11, 12, 13, and 14, reading the thousands and hundreds together as hundreds.

Thus, 6709, or sixty-seven hundred nine.

19. In 8888, for what does each of the 8's stand?

20. What effect would it have upon the value of 7 in 71, if a cipher were placed to the right of the 1?

21. When we reach a thousand, we begin to count the thousands as we did the units, thus: 1 thousand, 2 thousand, etc., up to 999 thousand, and when we have a thousand thousand, we call the number **one million**.

A thousand million we call a **billion**.

A thousand billion we call a **trillion**.

22. For convenience, when more than three figures are employed to express any whole number, they are divided into groups called **Periods**, the first of which, counting from the right, is used to denote any number from 1 to 999 *units*; the second, from 1 to 999 *thousands*; the third, from 1 to 999 *millions*, etc.

For convenience in reading, the periods are usually separated from each other by commas.

TABLE.

Hundreds of Tens of Units of	Hundreds of Tens of Units of	Hundreds of Tens of Units of	Hundreds of Tens of Units of	Hundreds of Tens of Units of
TRILLIONS.	BILLIONS.	MILLIONS.	THOUSANDS.	UNITS.
743,	986,	543,	206,	719
203,	777,	450,	810,	203
	25,	687,	100,	721
4,	400,	400,	496,	900
46,	785,	000,	010,	003
796,	045,	870,	903,	429

The first number in the table is read: *seven hundred forty-three trillion, nine hundred eighty-six billion, five hundred forty-three million, two hundred six thousand, seven hundred nineteen.*

Read the foregoing numbers, looking at the headings until you become familiar with the names of the periods.

23. *When we read numbers, we begin at the right and separate the numbers into periods of three figures each. Then, beginning at the left, we read each period as if it stood alone, adding its name.*

Each period, except the highest or left-hand period, must contain three figures.

EXERCISES.

24. Write and read :

1.	2.	3.	4.
700,000	903,744	300,300	17,019
965,237	200,080	37,961	586,240
83,016	147,906	145,808	770,300
728,000	400,004	307,029	562,999
5.	6.	7.	
1,789,645	769,834,521	211,467,431	
29,783,240	888,971,646	462,293,070	

8. Erase the figure at the right hand of each number in 5, 6, and 7, arrange the remaining figures in periods anew, by placing commas where they belong, and then read the numbers.

9. Continue thus with the given numbers, erasing the figures one by one, pointing off into periods correctly, and reading the numbers.

10. Make the following numbers larger by one thousand. By three thousand. By seven thousand. By nine thousand.

763

265

110

703

11. What number is next greater than 1000 ? 1699 ?
2019 ? 5082 ? 7200 ? 1239 ? 7379 ? 6299 ? 9999 ?
7676 ? 8109 ? 4099 ?

12. What number comes next before 3000 ? 6000 ? 5400 ?
2040 ? 2010 ? 5790 ? 1000 ? 2010 ? 9801 ?

13. How many hundreds and tens are there in 34 tens ?
In 86 tens ? In 37 tens ? In 42 tens ? In 128 tens ? In
564 tens ?

14. How many hundreds are there in 386 units ? In 864
units ? In 130 units ? In 175 units ? In 679 units ? In
534 tens ? In 643 tens ? In 161 tens ? In 879 tens ?

15. How many thousands and hundreds are there in 75
hundreds ? In 51 hundreds ? In 42 hundreds ? In 78 hun-
dreds ? How many in 250 tens ? In 175 tens ? In 549
tens ? In 543 tens ? How many in 9875 units ?

16. Make the following numbers larger, first by ten thou-
sand ; then by fifty thousand ; then by seventy thousand :

834 227 386 700 652 200 507 993

17. What number is next greater than 15,999 ? 150,109 ?
199,999 ? 777,779 ? 976,190 ? 275,300 ? 789,999 ?

18. What number is next less than 400,001 ? 600,000 ?
289,223 ? 500,000 ? 943,000 ? 727,800 ? 482,176 ?
100,000 ? 400,680 ? 400,999 ? 700,010 ? 300,000 ?

WRITTEN EXERCISES.

25. Write in figures :

1. Six thousand, eight hundred forty.
2. Eight thousand, two hundred fifty-seven.
3. Nineteen thousand, five hundred twenty-seven.
4. Eighty-four thousand, six hundred thirty-one.
5. One hundred thousand, seven hundred ten.
6. Fifty-eight thousand, six hundred one.
7. Two hundred thousand, four hundred seventy-five.
8. Four hundred eighteen thousand, two hundred seven.

9. Five hundred seven thousand, thirty.
10. Seven hundred fifteen thousand, nine hundred twenty-four.
11. Thirty-two million, four hundred fifteen thousand, two hundred thirty-three.
12. One hundred six million, eight hundred nine thousand, seven hundred fifty.
13. Two hundred sixty-five million, fifteen thousand, one hundred seventy-nine.
14. Three hundred sixty-five million, forty-nine.
15. Nine million, nine thousand, nine.
16. Eighty million, eighty thousand, eighty.
17. Six hundred million, six hundred thousand, six hundred.
18. Thirty-six million, eighteen thousand.
19. Twenty-four million, eight thousand, eight.
20. Two hundred million, two thousand, two.
21. Thirty-nine billion, sixty million, sixty thousand, sixty.
22. Fifty-two billion, one million, one thousand, one.
23. Sixty billion, sixty million, sixty thousand, sixty.
24. Eight hundred million, eighty-four thousand.
25. Seventy-seven billion, seventy-seven million, seventy-seven thousand, seventy-seven.
26. Four hundred twenty-six million, four hundred thousand, twenty-five.
27. One hundred seventy million, seven.
28. Sixteen hundred seventy-eight.
29. One trillion, seventeen million.
30. Four hundred fifty-six million, ten.
31. Five hundred nine million, four hundred thousand, twenty-five.
32. Nineteen hundred two.
33. One billion, six hundred thousand, four.

NOTATION AND NUMERATION OF UNITED STATES MONEY.

26. Write the names of all the United States coins you have seen, and tell of what each is made. What paper money have you seen?

In the currency of the United States,

10 mills make 1 cent.
100 cents make 1 dollar.

27. The Sign of Dollars is \$. It is written before the number.

Thus, \$27 is read, twenty-seven dollars.

28. In writing *cents* and *mills*, a period called the **Decimal Point** is placed before the number of cents.

29. *Cents* occupy the *first two* places at the right of the decimal point, and *mills* the *third* place.

Thus, \$0.35 is read, thirty-five cents; \$2.875 is read, two dollars eighty-seven cents five mills.

Mills are not coined. In business five mills or more are called one cent, and less than five are disregarded.

30. If the number of cents is less than ten, a cipher must be written in the first place at the right of the decimal point.

Thus, five cents is written \$.05; three cents, \$.03; seven dollars two cents five mills is written \$7.025.

EXERCISES.

31. Read the following:

1.	2.	3.	4.	5.
\$1.00	\$1.75	\$1.65	\$3.185	\$0.25.
\$4.87	\$18.05	\$15.43	\$70.10	\$1.05.
\$3.04	\$24.16	\$100.50	\$9.125	\$.40.
\$7.	\$2.357	\$10.	\$0.375	\$7.20.

Write the following in columns, keeping decimal points in a vertical line :

1. Eighteen dollars twenty-four cents.
2. Twenty-six dollars thirty-six cents.
3. Fifty-one dollars five cents five mills.
4. Thirty-eight dollars sixteen cents three mills.
5. Thirty-nine dollars nine cents.
6. Thirty dollars five cents.
7. One hundred dollars ten cents four mills.
8. Six dollars six cents.
9. Write one cent, two cents, three cents, etc., to fifty cents.
10. How many cents are there in \$1? In \$2? In \$3? In \$4? In \$8? In \$5? In \$9? In \$7? In \$6?
11. Read as cents: \$2; \$2.02; \$3; \$3.05; \$3.75; \$4.17; \$9.08; \$11.11.
12. Write the following as cents, using the character ¢ for cents :

\$1.25	\$1.10	\$3.75	\$7.45	\$9.67	\$8.80
\$7.05	\$9.81	\$4.13	\$4.18	\$5.56	\$6.72
13. Read the following as dollars and cents :
Thus, 1728¢ = 17 hundred 28 cents, or 17 dollars 28 cents.

2375¢	300¢	1015¢	1005¢	680¢	2793¢
1390¢	970¢	700¢	57345¢	3250¢	1809¢
14. Write the same, using the sign of dollars.
15. How many mills are there in 2¢? In 5¢? In 7¢? In 9¢? In 10¢? In 70¢? In 30¢? In 90¢? In 15¢?
16. How many mills are there in 100¢? In 300¢? In \$1? In \$2? In \$9? In \$1.20? In \$1.50?
17. Read the following as dollars, cents, and mills :

2765 mills	5639 mills	2793 mills
235 mills	9050 mills	4505 mills
18. Write the above, using the dollar sign.

ROMAN NOTATION.

32. The method of expressing numbers by means of letters is called **Roman Notation**.

It is called Roman Notation because it was originally used by the ancient Romans.

33. In this system, seven capital letters are used to express numbers, viz. :

LETTERS:	I,	V,	X,	L,	C,	D,	M.
VALUES:	1,	5,	10,	50,	100,	500,	1000.

By combining these letters according to certain principles, any number can be expressed.

34. PRINCIPLES. — 1. *When a letter is repeated, its value is repeated.*

Thus, X represents ten; XX, twenty; XXX, thirty; CC, two hundred; MM, two thousand.

2. *When a letter is placed before another of greater value, its value is to be taken from that of the greater.*

Thus, I represents one, V, five, and X, ten. But IV represents four; IX, nine; XL, forty; XC, ninety.

3. *When a letter is placed after another of greater value, their values are to be united.*

Thus, XV represents fifteen; LXX, seventy; LXXX, eighty; MC, one thousand, one hundred.

4. *When a letter is placed between two letters, each of greater value than itself, its value is to be taken from the sum of the other two.*

Thus, XIV represents fourteen; XIX, nineteen; CIV, one hundred four.

5. *A bar placed over a letter increases its value a thousand fold.*

Thus, V represents five; \overline{V} , five thousand; LXX represents seventy; \overline{LXX} , seventy thousand.

TABLE.

I = 1	XIII = 13	LX = 60
II = 2	XIV = 14	LXX = 70
III = 3	XV = 15	XC = 90
IV = 4	XVI = 16	C = 100
V = 5	XIX = 19	CC = 200
VI = 6	XX = 20	CCCC = 400
VII = 7	XXI = 21	CD = 400
VIII = 8	XXIX = 29	D = 500
IX = 9	XXX = 30	DCC = 700
X = 10	XXXIV = 34	M = 1000
XI = 11	XL = 40	MMM = 3000
XII = 12	L = 50	MDCCCXCH = 1892

EXERCISES.

Read the following:

XX	XIX	CDXX	XL	XCIV
LXX	XLIV	XXV	XC	<u>XIX</u>
XCIX	XXXV	LXIV	<u>VIII</u>	MDLIV
XXI	LXIX	CCXXIV	LX	MDCCC
XLIX	XXXVI	CCCLIX	DCXL	MMDC
MDCCCX	CCXXVI	DLXXI	MDXL	DCCL
CXCV	CCXLIV	MMMD	<u>IV</u> DXL	CCXCV

Express the following by Roman Notation :

23	61	84	35	312	517	1010	1900
34	19	59	47	419	493	1800	8000
15	36	62	86	226	499	1492	9000
27	43	97	214	384	278	1607	1721

ADDITION.

35. 1. How many blocks are 5 blocks, 3 blocks, and 7 blocks ?

2. How many splints are 3 splints, 2 splints, and 8 splints ?

3. How many oranges are 7 oranges, 2 oranges, and 5 oranges ?

4. How many are 5 and 6 and 7 ?

5. How many are 7 and 8 and 6 ?

6. How many are 5 and 9 and 2 and 6 ?

7. How many are 3 and 7 and 4 and 8 ?

8. How many are 4 and 8 and 5 and 9 ?

9. What have you been doing with the numbers given above ?

36. The process of finding a number that is equal to two or more given numbers is called **Addition**.

37. The result obtained by adding is the **Sum**, or **Amount**.

38. The **Sign of Addition** is a small upright cross (+). It is read *plus*, and is placed between the numbers to be added.

Thus, $4 + 2$ is read 4 plus 2, and means that 4 and 2 are to be added.

39. The **Sign of Equality** is two short parallel horizontal lines (=). It is read *equals*, or *is equal to*.

Thus, $3 + 2 = 5$ is read 3 plus 2 equals 5.

WRITTEN EXERCISES.

40. 1. Add 13, 12, 24, 30, and 14.

13 EXPLANATION. — In adding, the numbers are written in
 12 columns, units under units, tens under tens, and it is more
 24 convenient to begin with the units' column. Adding, we find
 30 there are 13 units. 13 means 1 ten and 3 units; therefore the 3 is
 14 written in the units' column, and the 1 ten is added to the tens.
 93 Adding the tens, we find that there are 9 tens. Therefore the
 result is 93.

In adding, do not say 4 and 4 are eight and 2 are ten,
 etc., simply name results, thus: 4, 8, 10, 13.

Find the sum of

2.	3.	4.	5.	6.	7.	8.	9.
72	73	84	57	\$ 27	\$ 21	\$ 96	\$ 20
85	97	17	46	72	53	81	38
90	16	52	93	18	38	45	47
<u>36</u>	<u>68</u>	<u>78</u>	<u>16</u>	<u>19</u>	<u>73</u>	<u>78</u>	<u>93</u>

10.	11.	12.	13.	14.	15.	16.	17.
39	54	89	28	59	25	45	68
29	90	24	77	83	27	17	39
72	71	74	50	17	48	18	74
45	29	93	33	26	16	13	56
<u>13</u>	<u>36</u>	<u>35</u>	<u>86</u>	<u>28</u>	<u>36</u>	<u>14</u>	<u>23</u>

18.	19.	20.	21.	22.	23.	24.	25.
45	62	42	65	43	68	56	42
58	87	24	89	69	42	49	46
34	49	85	38	22	25	17	34
22	36	48	76	85	34	21	58
<u>67</u>	<u>55</u>	<u>34</u>	<u>45</u>	<u>74</u>	<u>28</u>	<u>36</u>	<u>91</u>

26. Two birds fly in opposite directions, one 19 miles, and the other 27 miles. How far apart are they ?

27. Max earned 24 dollars, and John earned 9 dollars more than Max. How many dollars did both earn ?

28. Baby is 4 years old ; mamma is seven times as old as baby ; grandma is twice as old as mamma. What is the sum of their ages ?

29. In a store window I saw 38 books, 25 toys, 17 slates, and 20 globes. How many articles did I see ?

30. In one class in our school there are 11 boys and 9 girls ; in another, 17 boys and 18 girls ; in another, 10 boys and 14 girls. How many boys are there in all ? How many girls ? How many children ?

31. A fisherman caught 46 trout, 75 bass, and 13 blue-fish. How many fishes did he catch ?

32. Fred weighs 83 pounds, and Nathan weighs 97 pounds. How much do they both weigh ?

33. In one book there are 98 pages, in another 89, and in a third 75. How many pages are there in all the books ?

34. A certain tract of land was divided into 4 farms ; one containing 13 acres, another 37 acres, the third 70 acres, and the fourth 48 acres. How many acres did the original tract contain ?

35. The distance from Alburn to Grade is 6 miles, from Grade to Newton 8 miles, from Newton to Arnold 9 miles, and from Arnold to Houston 8 miles. How far is it from Alburn to Houston ?

36. Elsie solved 18 examples on Monday, 14 on Tuesday, and 12 on Wednesday. How many examples did she solve in all ?

37. If I buy a collar for 25 cents, some gloves for 37 cents, some ribbon for 8 cents, and a paper of needles for 9 cents, what do they all cost ?

38. A farmer has in one bin 28 bushels of oats, in another

36, in another 23. How many bushels has he in the three bins ?

39. If I sell 8 chickens to one man, 18 to another, and have 24 left, how many had I at first ?

40. Mr. Atwater bought a cow for 36 dollars, a sheep for 8 dollars, and a calf for 13 dollars. How much did he pay for all ?

41. Susie has 38 cents, and her brother has 17 cents more than she has. How many cents have they both ?

42. Morton's geography cost 75 cents, his reader 39 cents, and his slate 18 cents. How much did they all cost ?

43. Mr. Barnes has 3 fields. In one he put 36 cows, in another 42 cows, and in the third 27 cows. How many cows has he in the three fields ?

44. A fisherman sold 4 fishes for the following sums : the first for 37 cents, the second for 44 cents, the third for 54 cents, the fourth for 48 cents. What did he receive for all ?

45. On a Christmas tree I counted 12 dolls, 25 balls, 28 flags, 5 whistles, 3 drums, 25 candy-bags, and 14 books. How many things did I count ?

46. Thornton has 68 marbles, Oliver has 10 less than Thornton. How many have both boys together ?

47. One stalk of a plant has 43 leaves, a second stalk has 36 leaves, and a third has 18 leaves. How many leaves have they all ?

48. A baker made at one time 37 loaves, at another 56, at another 28. How many loaves did he make in all ?

49. A milkman sold 72 quarts of milk on Monday, 56 on Tuesday, 85 on Wednesday, and 49 on Thursday. How many quarts did he sell in the four days ?

50. A butcher sold 24 pounds of veal, 37 pounds of pork, 38 pounds of chickens, and 49 pounds of beef. How many pounds of meat did he sell ?

51. A man has 45 sheep, 28 cows, and 15 calves. How many animals has he?

52. If I spend 8 dollars a week for board, 3 dollars for car fare, and have 14 dollars left of my week's earnings, how much did I earn?

53. Wilfred paid 28 dollars for a desk. He paid 5 dollars to have it polished, and then sold it for 7 dollars more than it cost him. How much did he get for the desk?

54. Commencing with 28, find the sum of 28 and the succeeding numbers less than 36?

55. A cloth merchant sold 5 rolls of cloth, containing respectively 46, 52, 63, 29, and 76 yards. How many yards did he sell?

56. A saddle cost 14 dollars, a wagon 60 dollars more than the saddle, and a horse as much as the saddle and wagon together. How much did the horse cost? How much did all cost?

57. Harold earned 87 cents. If he previously had 36 cents, and then found 7 cents, how many cents had he in all?

58. Howard wrote 38 lines of German on Monday, 27 on Tuesday, 15 on Wednesday, 17 on Thursday, and 26 on Friday. How many lines did he write during the week?

59. A man sold three lots. For one he received \$695, for another \$734, and for the third \$392. How much did he receive for all?

60. A owns a farm of 518 acres, B one of 425 acres, and C one of 392 acres. How many acres do they all own?

61. In a city there were four schools, the first containing 396 pupils, the second 683 pupils, the third 718 pupils, and the fourth as many as the second and third together. How many pupils attended the schools? How many attended the fourth school?

Add the following:

62.	63.	64.	65.	66.	67.	68.
68	54	37	81	39	55	38
45	32	93	39	23	38	43
93	81	86	44	87	37	71
87	36	94	87	65	25	64
94	57	83	39	34	61	83
86	91	27	86	56	39	73
83	87	55	54	83	24	98
<u>37</u>	<u>32</u>	<u>66</u>	<u>37</u>	<u>74</u>	<u>86</u>	<u>72</u>

69.	70.	71.	72.	73.	74.
452	374	418	519	735	875
378	516	372	327	872	912
684	834	384	684	659	386
359	912	596	519	738	596
871	816	387	329	486	837
964	325	426	548	684	542
832	426	385	819	396	816
<u>576</u>	<u>845</u>	<u>713</u>	<u>384</u>	<u>845</u>	<u>347</u>

75.	76.	77.	78.	79.
3785	7351	7638	6835	381
4651	4286	1842	7867	3468
3278	3875	3451	392	193
5964	1894	4265	8437	8200
8137	5768	7169	24	57
8462	3681	8376	6174	93
5738	5437	1452	5762	3956
9124	8796	3814	8600	4184
6783	5106	6842	439	324
3854	7340	6937	3764	47
4265	4165	5843	158	596
<u>7384</u>	<u>9354</u>	<u>7165</u>	<u>99</u>	<u>1938</u>

SUBTRACTION.

41. 1. How many cents has George if he lost 5 cents of the 9 cents which he had ?

2. Ten horses were drawing wagons, all but 5 of which were gray. How many gray horses were there ?

3. Bertha had 13 cents and spent 6 cents. How much had she left ?

4. How many are 10 less 3 ? 15 less 8 ? 17 less 7 ?

5. How many are 16 less 7 ? 19 less 9 ? 13 less 5 ?

6. How many are 12 less 8 ? 14 less 5 ? 15 less 7 ?

7. How many are 13 less 6 ? 17 less 9 ? 18 less 9 ?

8. What have you been doing with the above numbers ?

42. The process of finding the difference between two numbers, or of taking part of a number from it and finding how many are left, is called **Subtraction**.

43. The number from which another is taken is called the **Minuend**.

44. The number to be subtracted is the **Subtrahend**.

45. The number remaining after one number is taken from another is called the **Remainder**.

The remainder is the difference between the *minuend* and the *subtrahend*.

46. The **Sign of Subtraction** is a short horizontal line (—). It is called **Minus**.

Thus, $8 - 3$ is read, 8 minus 3, and means that 3 is to be subtracted from 8.

WRITTEN EXERCISES.

47. 1. From 869 subtract 423.

Minuend	869	EXPLANATION. — For convenience, the less
Subtrahend	423	number is written under the greater, units under
Remainder	446	units, tens under tens, etc. Beginning at the right,
		each order of units in the subtrahend is subtracted
		separately from the same order in the minuend.

Thus, 9 units — 3 units are 6 units, which are written under units.

6 tens — 2 tens are 4 tens, which are written under the tens.

8 hundreds — 4 hundreds are 4 hundreds, which are written under the hundreds. Hence the remainder is 446.

PROOF. — 446, the remainder, plus 423, the subtrahend, equals 869, the minuend. Hence the result is correct.

Copy, subtract, and prove:

2.	3.	4.	5.	6.	7.	8.
888	999	564	899	329	957	864
356	276	213	486	115	516	231
9.	10.	11.	12.	13.	14.	15.
435	768	975	543	879	768	599
234	253	432	133	217	642	143
	16.	17.	18.	19.	20.	
From	876	789	968	999	888	
Take	242	176	723	275	234	

21. Susie had two bundles of 10 splints each, and 6 more. How many splints had she? She gave her teacher 8 of the splints. How many had she left?

116	She found the 6 ones were not enough to give
26	her teacher, so she took one bundle of the tens,
8	untied it, and put it with her 6 ones, making 16
	ones besides the 1 ten which she had not untied.
18, Remainder.	She took 8 away from the 16 and found she had 8
	left, and also the 1 ten, or 18 left in all.

Do just as Susie did.

22. Take 1 bundle of one hundred splints, 4 bundles of 10 splints each, and 5 single splints. How many have you in all? Give your teacher 27 of the splints. Compare what you did with the following solution:

$$\begin{array}{r} 3 \text{ } 15 \\ 145 \\ 27 \\ \hline \end{array}$$

118, Remainder.

23. Annie had 2 bundles of one hundred splints each. She gave Fred 78. How many had she left?

Annie took 1 bundle of one hundred, untied it, and found there were 10 tens in it. She left 9 of the tens a moment and untied 1 bundle of 10, finding 10 ones in it. Of these she gave 8 to Fred, and had 2 left. Then taking up the 9 tens, she gave 7 tens to Fred, and had 2 tens left, and besides these the 1 bundle of one hundred. She had, therefore, 122 left.

Find the difference between the following, and supply the numbers omitted in minuend, subtrahend, or remainder:

24.	25.	26.	27.	28.	29.	30.	31.
<u>34</u>	<u>86</u>	<u>92</u>	<u>41</u>	<u>70</u>	<u>83</u>	<u>44</u>	<u>96</u>
<u>17</u>	<u>69</u>	<u>55</u>	<u>18</u>	<u>29</u>	<u>45</u>	<u>28</u>	<u>73</u>
32.	33.	34.	35.	36.	37.	38.	39.
<u>99</u>	<u>72</u>	<u>77</u>	<u>77</u>	<u>91</u>	<u>85</u>	<u>88</u>	<u>65</u>
<u>45</u>	<u>48</u>	<u>43</u>	<u>29</u>	<u>88</u>	<u>17</u>	<u>33</u>	<u>28</u>
40.	41.	42.	43.	44.	45.	46.	47.
<u>871</u>	<u>761</u>	<u>240</u>	<u>166</u>	<u>752</u>	<u>631</u>	<u>382</u>	<u>546</u>
<u>257</u>	<u>347</u>	<u>18</u>	<u>139</u>	<u>239</u>	<u>408</u>	<u>272</u>	<u>338</u>
	48.	49.	50.	51.	52.	53.	54.
Minuend,	<u>300</u>	<u>204</u>	<u>500</u>	<u>607</u>	<u>908</u>	<u>550</u>	<u>600</u>
Subtrahend,	<u>171</u>	<u>78</u>	<u>183</u>	<u>179</u>	<u>127</u>	<u>258</u>	<u>379</u>
Remainder,							

	55.	56.	57.	58.	59.	60.	61.
Minuend,	635	591	624	350	870	980	527
Subtrahend,	<u>469</u>	<u>268</u>	<u>156</u>	<u>123</u>	<u>456</u>	<u>379</u>	<u>318</u>

Remainder,

	62.	63.	64.	65.	66.	67.
Minuend,	?	769	?	780	800	?
Subtrahend,	<u>17</u>	<u>?</u>	<u>238</u>	<u>?</u>	<u>?</u>	<u>502</u>
Remainder,	136	144	196	123	191	139

	68.	69.	70.
Minuend,	\$ 541	\$ 905	768 children.
Subtrahend,	<u>\$ 492</u>	<u>\$ 184</u>	<u>599</u> "
Remainder,	\$	\$	children.

	71.	72.	73.
Minuend,	808 eggs.	967 birds.	842 pints.
Subtrahend,	<u>527</u> "	<u>209</u> "	<u>763</u> "
Remainder,	eggs.	birds.	pints.

	74.	75.	76.
Minuend,	400 horses.	502 cows.	\$ 550
Subtrahend,	<u>285</u> "	<u>237</u> "	<u>\$ 118</u>
Remainder,	horses.	cows.	\$

	77.	78.	79.
Minuend,	800 bushels.	? apples.	? days.
Subtrahend,	<u>269</u> "	<u>372</u> "	<u>18</u> "
Remainder,	bushels.	263 apples.	347 days.

	80.	81.	82.
Minuend,	75 years.	\$ 772	\$ 324
Subtrahend,	<u>?</u> "	<u>\$ 624</u>	<u>?</u>
Remainder,	19 years.	\$	\$ 136

REVIEW EXERCISES.

ADDITION AND SUBTRACTION.

48. 1. From a tank containing 935 gallons of water, 648 gallons were drawn off, after which 247 gallons ran in. How many gallons were there then in the tank ?

2. Two trains start from Chicago. One goes east 298 miles, and the other west 247 miles. How far apart are they ?

3. A man purchased 8983 bricks, but used only 5362. How many had he left ?

4. A man paid \$465 for a carriage. He spent \$57 for repairs on it, and sold it for \$500. Did he gain or lose, and how much ?

5. From the sum of 763 and 297 subtract their difference.

6. A horse and cow together cost \$276. If the cow cost \$81, what was the cost of the horse ?

7. Two trains start from cities 582 miles apart. If they travel towards each other, one 123 miles and the other 238 miles, how far apart are they ?

8. Columbus discovered America in 1492. How many years have passed since then ?

9. A merchant bought 30 pieces of cloth, containing in all 979 yards. He afterwards sold all but 143 yards. How many yards did he sell ?

10. The cost of building a house was \$3972, and of furnishing it \$1597. What was the entire cost ? How much more did it cost to build it than to furnish it ?

11. Cotton was first planted in the United States about 1759. How long ago was that ?

12. Ralph threw a ball 37 feet, and Jamie threw his 19 feet farther than Ralph. How many feet must Jamie go to pick up his ball and return ?

13. A man owned a horse that cost \$852, and a carriage that cost \$437. He was obliged to sell them both for \$1000. How much money did he lose by the sale?

14. My neighbor has in his orchard 264 pear trees, which is 187 less than the number of his apple trees. How many apple trees has he? How many trees has he in all?

15. A farmer having 425 bushels of wheat, sold 127 bushels to one person, and 264 bushels to another. How many bushels had he left?

16. $\$24.03 + \$17.75 + \$19.63 + \$10 + \$18.09 + \$7.99 = ?$

17. $\$10 - (\$2.35 + \$3.78 + \$1) = ?$

18. Mr. Jones earned during July \$172. He paid a bill for \$28 to the grocer, \$19 to the butcher, and \$36 for rent. How much of the money remained?

19. $\$4700 - (\$462 + \$2180) = ?$

$\$3900 - (\$2614 + \$854.50) = ?$

20. A man gave to his son \$575; to his daughter, \$468; and to his nephew, \$249 less than to his daughter. How much did he give to all?

21. Two men, who are 425 miles apart, travel towards each other, one 36 miles, the other 58 miles a day. How far apart will they be at the end of two days?

22. If the same men travel away from each other, how far apart will they be at the end of two days?

23. In a certain school there are 692 pupils. In the primary grade there are 325; in the intermediate grade, 259; and the remainder are in the grammar grade. How many are in the grammar grade?

24. A man bought a lot of land for \$1860, and built upon it a house costing \$7584. If he sold the property for \$10,000, how much did he make?

25. If I buy at a grocery store sugar for 78 cents, cheese for 39 cents, and nuts for 25 cents, giving in payment a \$2 bill, how much change should I receive?

26. From 946 subtract 486; from the result subtract 284; to this add 398; and from this answer subtract 199.

27. Lena's book has 482 pages. If she reads 168 pages in the morning, and 89 pages in the afternoon, how many more pages has she to read?

28. Bert and Harry had each \$1.40. Bert earned 28¢ more, and Harry spent 72¢ of his money. How much more has Bert than Harry?

29. In a school there were 278 girls and 215 boys. If 29 girls and 34 boys leave, how many pupils will there then be attending the school?

30. Into a bin that holds 825 bushels, I put 272 bushels of corn, and afterwards 346 bushels more. How many more bushels of corn will it hold?

31. If I buy a hat for \$6.75 and give in payment a \$10 bill, what change should I receive?

32. The entire expenses of a journey made by a family were \$391.86. The fare on the railroads was \$192.15, carriage hire was \$37.25, fare on the steamers amounted to \$61.35, and the rest was paid for board. What did the board cost?

33. A merchant failed in business, owing debts to the amount of \$1837.85. His goods on hand were worth \$894.50, and there was due him from persons who had bought goods from him, \$591.75. How much more did he owe than he could pay?

34. $\$68.95 + \$81.36 - \$29.15 - \$42.36 + \$25.84 = ?$

35. $\$76.37 + \$34.93 + \$8.83 - \$16.25 - \$13.24 = ?$

36. $\$59.23 + \$18.76 - \$32.17 + \$7.08 - \$41.25 = ?$

37. $\$32.94 - \$16.23 + \$13.81 - \$20.06 + \$24.53 = ?$

38. $\$68.92 + \$23.75 - \$15.27 - \$6.35 + \$15.18 = ?$

39. $\$37.50 - \$18.24 + \$60.08 - \$18.32 - \$16.29 = ?$

40. $\$51.23 + \$67. \quad + \$59.38 - \$62.81 - \$4.85 = ?$

41. $\$24.69 + \$41.80 - \$23 + \$27.42 - \$8 + \$15 = ?$

MULTIPLICATION.

49. 1. Henry worked 5 days per week for 5 weeks. How many days did he work in that time?

2. A boy delivered books to 12 houses, leaving 6 books at each house. How many books did he carry altogether?

3. The wages of a laborer were \$12 per week. How much could he earn in 6 weeks?

4. The price of a cord of wood is \$5. How much must be paid for 7 cords?

5. The postman who delivers our mail travels 9 miles per day. How far will he travel in 8 days?

6. A lady bought 11 yards of cambric at 10 cents per yard. How much did it cost her?

7. A good walker can walk 4 miles per hour. How far can he walk in 9 hours at that rate?

8. How many are eight 3's? Nine 5's?

9. How many are seven 7's? Eight 6's?

10. How many are six 8's? Nine 4's?

11. How many are four 6's? Five 4's?

12. What have you been doing with the numbers in the examples given above?

50. A short process of finding the sum of several equal numbers, or the process of taking one number as many times as there are units in another is called **Multiplication**.

51. The number taken or multiplied is called the **Multiplicand**.

52. The number which shows how many times the multiplicand is taken is called the **Multiplier**.

53. The result obtained by multiplying is called the **Product**.

54. The multiplicand and multiplier are called the **Factors** of the product.

55. The **Sign of Multiplication** is an oblique cross, \times . It is read *multiplied by* when the multiplicand precedes it and *times* when the multiplier precedes it.

Thus 6×8 is read 6 multiplied by 8 when 6 is the multiplicand, but it is read 6 times 8 when 6 is the multiplier.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXPLANATION.—The numbers in the left-hand column may be regarded as the multipliers, and the numbers across the top as the multiplicands. The products will be found in the *horizontal columns* opposite the multipliers.

Thus, 2 ones are 2 ; 2 twos are 4 ; 2 threes are 6 ; 2 fours are 8, etc.

The order may be changed so that the numbers in the upper horizontal line may be regarded as the multipliers, and the numbers on the left as the multiplicands. Each successive multiplier may be used with each multiplicand.

Thus, 1 two is 2 ; 2 twos are 4 ; 3 twos are 6 ; 4 twos are 8, etc.

DRILL EXERCISES.

Tell instantly the following products:

8×4	5×11	12×6	7×5	2×7
5×3	12×7	11×5	2×6	3×4
6×8	9×9	4×9	3×5	2×4
7×2	8×12	5×5	4×2	12×3
9×8	3×3	7×6	5×2	9×4
8×7	4×10	9×7	6×9	10×12
3×9	5×7	10×8	7×11	11×4
4×6	6×5	11×10	7×3	2×3
5×6	11×7	2×11	2×9	8×6
3×7	12×9	9×2	4×4	6×7
4×8	7×8	5×9	6×3	12×8
5×12	9×6	7×4	10×11	10×4
6×11	10×10	9×11	8×11	8×10
7×10	11×11	10×7	7×12	4×7
8×9	12×12	8×8	9×10	10×3
9×5	7×7	4×5	10×6	12×2
5×8	8×5	5×10	11×9	4×12
3×10	2×8	6×12	12×11	3×2
4×3	2×12	7×9	2×5	11×3
3×11	9×3	6×10	3×8	11×2
2×10	3×12	6×6	6×2	11×6
5×4	8×3	12×5	10×5	12×10
10×9	3×6	8×2	11×12	10×2
11×8	6×4	9×12	12×4	4×11

56. When the multiplier is not greater than 12.

ORAL EXERCISES.

1. When oranges cost 4 cents apiece, how much must Mary pay for 6 oranges?

2. If Lulu changes 6 half-dimes into cents, how many cents will she have?

3. Henry went to school every school-day for 8 weeks. How many days was that?

4. A girl bought 9 yards of ribbon for 8 cents a yard? How much did it cost her?

5. An organ-grinder received 7 cents at each of 8 houses. How much did he get in all?

6. A boy sold 9 pencils at 4 cents apiece. How much did he get for them all?

7. In a school-room there were 8 rows of seats, and 8 seats in a row. How many pupils could be seated there?

8. A half-quire of paper contains 12 sheets. How many sheets are there in 6 half-quires?

9. A drawing book cost 12 cents. If each pupil in a class of 9 pupils paid the same price for a drawing book, how much did they all pay?

10. Harry had to pay 9 cents per day for his fare in going to and from school. How much was that per week?

11. Alice planted 9 rows of asters in her flower bed, and each row had 9 plants. How many asters did she set out?

12. Each pupil in a class wrote 7 lines in his writing book. If there were 7 pupils in the class, how many lines did they all write?

13. The cost of a railroad ticket from Ashburn to Graystone is 11 cents. How much will 8 such tickets cost?

14. Some children are required to attend school 6 hours per day. How many hours will they be at school in 7 days?

15. If Mary goes to bed at 8 o'clock at night, and rises at 7 o'clock in the morning, how many hours does she spend in bed in 9 days?

16. Reuben said that he saw 5 flocks of birds, each containing 7 birds. How many birds did he see?

17. A school week is 5 days. How many days are there in 9 school weeks?

18. Sarah made on her slate 9 rows of squares, each row containing 7 squares. How many squares did she make?

19. Henry brought home 5 dozen eggs. How many eggs did he bring?

20. Susie's father worked 9 hours per day for 7 days. How many hours did he work in all?

21. How many are two 2's? Three 2's? Four 2's? Five 2's? etc. Give the table of 2's.

22. Give the table of 3's. Of 4's. Of 5's. Of 6's. Of 7's. Of 8's. Of 9's. Of 10's. Of 11's. Of 12's.

23. Count by 5's from 0 to 100 in this way: 0, 5, 10, 15, 20, 25, 30, etc.

24. Count by 3's from 0 to 36. By 7's from 0 to 84.

25. Count by 6's from 0 to 72. By 9's from 0 to 108.

26. Count by 4's from 0 to 48. By 8's from 0 to 96.

27. Count by 10's from 0 to 110. By 12's from 0 to 144.

WRITTEN EXERCISES.

57. 1. How many are 6 times 397?

Multiplicand,	397	EXPLANATION. — For convenience in multiplying, the multiplier is written under the multiplicand and we begin at the right to multiply in the following manner:
Multiplier,	6	
Product,	2382	

6 times 7 units are 42 units. But 42 units are 4 tens and 2 units; therefore, the figure 2 is written in units' place in the product, and the 4 tens are reserved to add to the tens of the product.

6 times 9 tens are 54 tens, plus 4 tens reserved are 58 tens, or 5 hundreds and 8 tens; therefore the figure 8 is written in tens' place in

the product, and the 5 hundreds are reserved to add to the hundreds in the product.

6 times 3 hundreds are 18 hundreds, plus 5 hundreds reserved are 23 hundreds, or 2 thousands and 3 hundreds, which are written in thousands' and hundreds' places in the product. Hence the product is 2382.

Find the products of the following:

- | | | |
|---------------------|----------------------|-----------------------|
| 2. $5 \times 46.$ | 14. $415 \times 7.$ | 26. $3245 \times 4.$ |
| 3. $6 \times 37.$ | 15. $372 \times 8.$ | 27. $2861 \times 5.$ |
| 4. $7 \times 28.$ | 16. $846 \times 5.$ | 28. $3942 \times 3.$ |
| 5. $6 \times 93.$ | 17. $364 \times 4.$ | 29. $5687 \times 6.$ |
| 6. $7 \times 86.$ | 18. $918 \times 3.$ | 30. $3834 \times 8.$ |
| 7. $4 \times 98.$ | 19. $325 \times 5.$ | 31. $5279 \times 4.$ |
| 8. $3 \times 69.$ | 20. $864 \times 9.$ | 32. $8168 \times 9.$ |
| 9. $8 \times 58.$ | 21. $325 \times 4.$ | 33. $3934 \times 7.$ |
| 10. $9 \times 43.$ | 22. $816 \times 6.$ | 34. $5468 \times 5.$ |
| 11. $6 \times 85.$ | 23. $932 \times 7.$ | 35. $3173 \times 2.$ |
| 12. $11 \times 87.$ | 24. $519 \times 11.$ | 36. $4284 \times 11.$ |
| 13. $12 \times 92.$ | 25. $837 \times 12.$ | 37. $1987 \times 12.$ |

38. There are 5280 feet in a mile. How many feet does a boy walk who walks 5 miles?

39. A merchant sold Mrs. Allen 5 pieces of muslin, each containing 39 yards. How many yards did he sell to her?

40. The average wages of 5 clerks were \$37.50 per month. How much did they all earn in a month?

41. When hay sells at \$8.25 per ton, how much must I pay for 8 tons?

42. If a railway train runs 47 miles per hour, how far will it run in 8 hours?

43. Theodore Schuyler & Co. paid each of 5 bookkeepers \$85.25 per month. How much did they pay them all?

44. A laborer found that his expenses were \$23.85 per month. How much will they be in 8 months?

45. A carriage maker sold 8 carriages for \$78.75 apiece. How much did he receive for them all?

46. Mr. A loaned a man some money and received for the use of it, or interest, \$18.35 per year. How much should he receive for the use of it for 5 years?

47. A contractor built 8 houses for which he received \$7325 apiece. How much did he receive for all?

48. The expenses of a large dry-goods store were \$3215.25 per month. How much were they for 8 months?

49. An organ manufacturer sold during 1891 nine church organs, the average price of which was \$7963. How much did his sales amount to that year?

50. The number of papers sold by the publisher of a daily paper averaged 8432 each day. How many were sold in 6 days?

51. A mechanic earned \$87.35 per month for 8 months. How much were his entire earnings in that time?

52. A man sold 9 carriages for \$92.55 apiece. How much did he get for them?

53. The steamship Roduna consumes 125 tons of coal per day. How much will she consume in a voyage lasting 8 days?

54. In one of the cities of the United States, the average number of passengers carried upon the street-cars daily was 38,395. How many were carried in 6 days?

55. The sales of tickets at a railway station for the month of January amounted to \$38,495. If they are the same, on the average, for 7 months, how much will they be?

56. The interest paid for a sum of money was \$384.27 per year. How much interest money was paid in 5 years?

57. The increase in population in a city for the year 1891 was 13,868. At that rate, what will be the increase in 6 years?

58. The average distance sailed by a steamship each voy-

age between New York and Liverpool, England, was 2831 miles. How many miles did she sail in 4 round trips?

59. A gardener sold last year 9765 bunches of asparagus. How many bunches would he sell in 7 years at that rate?

Multiply each of the following numbers by 2, 3, 4, 5, 6, 7, 8, and 9:

60. 5869.	63. 3954.	66. 86,973.	69. 18,572.	72. 48,362.
61. 7296.	64. 3857.	67. 23,547.	70. 42,364.	73. 32,573.
62. 8164.	65. 5679.	68. 65,132.	71. 38,453.	74. 71,685.

58. When the multiplier is expressed by more than one figure.

ORAL EXERCISES.

1. How many are 10 times 6? 10 times 7? 10 times 8? 10 times 5? 10 times 9?

2. How many are 10 times 3 plus 3 times 3 or 13 times 3? 10 times 4 plus 6 times 4 or 16 times 4? 10 times 7 plus 3 times 7 or 13 times 7? 10 times 8 plus 5 times 8 or 15 times 8?

3. How many are 15 times 4? 18 times 3? 17 times 5? 16 times 6? 12 times 8? 14 times 3?

4. How many are 13 times 5? 13 times 6? 13 times 7?

5. What will 15 yards of ribbon cost at 5 cents per yard? At 8 cents per yard?

6. How many days are there in 17 weeks?

7. How many quarts are there in 16 gallons?

8. How many feet are there in 13 yards?

9. How much will 15 yards of calico cost at 6 cents per yard? At 9 cents per yard?

10. How much does a man save in 12 weeks if he saves \$8 per week?

11. There are 9 square feet in a square yard. How many square feet are there in 14 square yards?

12. How many are 10 times 9? 10 times 5? 10 times 7?
13. What figure must be annexed to 9 to multiply it by 10? What to 5? What to 7?
14. How, then, is any number multiplied by 10?
15. How many are 10 sevens or 10 times 7? 100 sevens or 100 times 7? 100 eights or 100 times 8? 100 times 6? 100 times 15? 100 times 35? 100 times 48? 100 times 56? 100 times 84? 100 times 97? 100 times 63?
16. How, then, is any number multiplied by 100?
17. How many are 100 times 9? 1000 times 9? 1000 times 7? 1000 times 15? 1000 times 35? 1000 times 43? 1000 times 54? 1000 times 75?
18. How, then, is any number multiplied by 1000?
19. Since a number is multiplied by 10 by annexing *one* cipher to the number; by 100 by annexing *two* ciphers; by 1000 by annexing *three* ciphers, how may a number be multiplied by 10, 100, 1000, etc., or by 1 with any number of ciphers annexed?
59. *A number is multiplied by 10, 100, 1000, etc., or by 1 with any number of ciphers annexed, by annexing to the multiplicand as many ciphers as there are in the multiplier.*

WRITTEN EXERCISES.

60. What are the products of the following:

- | | | |
|---------------------|------------------------|----------------------------|
| 1. 45×10 . | 6. 324×100 . | 11. 5463×1000 . |
| 2. 67×10 . | 7. 416×100 . | 12. 3784×1000 . |
| 3. 38×10 . | 8. 723×100 . | 13. 5169×10000 . |
| 4. 46×10 . | 9. 342×100 . | 14. 4237×10000 . |
| 5. 76×10 . | 10. 518×100 . | 15. 8193×100000 . |

16. Multiply 832×3000 .

$$\begin{array}{r} 832 \\ 3000 \\ \hline 2496000 \end{array}$$

EXPLANATION. — Since 3000 is 1000 times 3, 832 is first multiplied by 3, giving 2496 as a product, and this product is multiplied by 1000 by annexing three ciphers.

17. 45×20 .	27. 346×500 .	37. 3946×6000 .
18. 37×30 .	28. 279×300 .	38. 5279×4000 .
19. 68×40 .	29. 518×700 .	39. 8365×7000 .
20. 73×80 .	30. 264×900 .	40. 7226×9000 .
21. 56×60 .	31. 385×400 .	41. 5387×8000 .
22. 90×40 .	32. 498×600 .	42. 7394×5000 .
23. 30×50 .	33. 341×800 .	43. 2869×3000 .
24. 23×90 .	34. 957×200 .	44. 9999×2000 .
25. 84×70 .	35. 691×500 .	45. 8765×4000 .
26. 65×60 .	36. 444×400 .	46. 9182×6000 .

47. Multiply 373×235 .

FIRST PROCESS.	373	SECOND PROCESS.	373
	<u>235</u>		<u>235</u>
1st partial product,	1865		1865
2d partial product,	11190		1119
3d partial product,	<u>74600</u>		<u>746</u>
Entire product,	87655		87655

EXPLANATION OF FIRST PROCESS. — For convenience, the multiplier is written under the multiplicand with units under units, tens under tens, etc.

Since we cannot multiply by 235 at one operation, we multiply by the parts of 235, viz. : 5 units, 3 tens or 30, and 2 hundreds or 200, and add the products.

5 times 373 is 1865, the first partial product ; 30 times 373 is 11190, the second partial product ; 200 times 373 is 74600, the third partial product. The sum of these partial products is 87655, the entire product.

EXPLANATION OF SECOND PROCESS. — The second process is the one commonly used. It is the same as the first except that the ciphers are omitted from the right of the partial products. The significant figures, however, occupy their proper places.

Thus, in multiplying by 3 tens the product is 1119 tens, and consequently, the right-hand figure of this product is placed under tens and the others in their proper places in the product.

In multiplying by 2 hundreds or any number of hundreds the lowest order of the product is hundreds, consequently the right-hand figure of the product is written under hundreds.

61. From these processes the following general method of multiplying is readily deduced :

1. *The multiplier is written under the multiplicand, units under units, tens under tens, etc.*

2. *Each figure of the multiplicand is multiplied by each significant figure of the multiplier and the right-hand figure of each product is placed under the figure of the multiplier used to obtain it.*

3. *The sum of the partial products will be the entire product.*

When there is a cipher in the multiplier, multiply by the significant figures only, taking care to place the right-hand figure of each partial product under the figure used to obtain it.

$$\begin{array}{r}
 48. \\
 5684 \\
 609 \\
 \hline
 51156 \\
 34104 \\
 \hline
 3461556
 \end{array}$$

$$\begin{array}{r}
 49. \\
 78645 \\
 5008 \\
 \hline
 629160 \\
 393225 \\
 \hline
 393854160
 \end{array}$$

$$\begin{array}{r}
 50. \\
 59658 \\
 3700 \\
 \hline
 417606 \\
 178974 \\
 \hline
 220734600
 \end{array}$$

Multiply :

51. 267 by 24.

66. 3845 by 236.

81. 5698 by 792.

52. 836 by 27.

67. 7186 by 314.

82. 3179 by 865.

53. 918 by 61.

68. 5327 by 420.

83. 8169 by 734.

54. 309 by 38.

69. 6284 by 518.

84. 5234 by 826.

55. 465 by 27.

70. 4769 by 344.

85. 3186 by 839.

56. 732 by 55.

71. 6873 by 530.

86. 5279 by 918.

57. 817 by 63.

72. 2918 by 364.

87. 3864 by 609.

58. 398 by 84.

73. 4873 by 273.

88. 38,675 by 708.

59. 426 by 29.

74. 9275 by 731.

89. 42,308 by 692.

60. 918 by 76.

75. 8462 by 820.

90. 75,069 by 804.

61. 837 by 84.

76. 7319 by 394.

91. 83,427 by 3625.

62. 695 by 32.

77. 8425 by 972.

92. 80,069 by 5846.

63. 864 by 69.

78. 6685 by 873.

93. 58,325 by 3192.

64. 528 by 87.

79. 6517 by 762.

94. 38,164 by 8068.

65. 364 by 44.

80. 7395 by 469.

95. 49,234 by 9096.

96. \$48.27 by 584.

101. \$543.79 by 3980.

97. \$39.35 by 376.

102. \$708.27 by 2900.

98. \$73.86 by 705.

103. \$496.83 by 5805.

99. \$85.18 by 835.

104. \$385.76 by 3760.

100. \$49.66 by 597.

105. \$483.74 by 5736.

106. A manufacturer sold 29 reapers for \$96.85 apiece. How much did he get for them?

107. A man walked 37 miles. How many feet did he walk, since there are 5280 feet in a mile?

108. A tract of land containing 294 acres was sold for \$313 per acre. For how much did it sell?

109. It required 318 loads of bricks averaging 1387 bricks to build a school building. How many bricks was that?

110. A railroad employed 213 switchmen, whose wages averaged \$78.34 per month. How much did all the switchmen receive per month?

111. A merchant sold 7 pieces of cloth, each containing 34 yards, for \$2.17 per yard. How much did he receive for it?

112. The Danton Leather Co. purchased 37 lots of hides, each lot containing 384 hides. How many did they purchase?

113. A cloth dealer sold 54 pieces of cassimere, each containing 48 yards, at \$1.87 per yard. How much did he receive for it?

114. A fire destroyed 345 gross of pencils. Since a gross is 144, how many pencils were destroyed?

115. In a day there are 1440 minutes. How many minutes are there in a year?

116. A farm of 95 acres was sold for \$37.25 per acre. How much was received for it?

117. If your pulse beats 4320 times per hour, how many times will it beat in 2 days of 24 hours each?

118. The cost of building a railroad was \$31,755 per mile.

If it was 118 miles long, how much did it cost to construct it?

119. A pump can pump 365,864 gallons per day. How many gallons can it pump in 31 days?

120. The expense of an army in Europe is, on the average, \$195 per man. How much will it cost to keep an army of 32,596 men?

121. If a locomotive costs \$19,325, how much will 23 such locomotives cost?

122. An army used 3285 barrels of flour each day for 49 days. How many barrels did it use altogether?

123. A shoe manufacturer in Massachusetts sold 23,876 pairs of shoes at an average price of \$3.15 a pair. How much did he receive for them?

REVIEW.

ORAL EXERCISES.

62. 1. Henry bought 3 pencils at 3 cents apiece, and 5 postage stamps at 2 cents apiece. How much did he pay for all?

2. The wages of a clerk were \$15 per week. If he paid \$5 for board and \$4 for other expenses, how much did he save in 5 weeks?

3. Nina spent 20 cents for a box of water-color paints, 5 cents for brushes, and had 10 cents left. How much money had she at first?

4. Two men travel in opposite directions from a starting-point, one at the rate of 5 miles per hour, and the other at the rate of 4 miles per hour. How far apart are they at the end of 5 hours?

5. Mary counted the petals on 6 flowers. All but 2 of them had 6 petals each, and the others had 5 each. How many petals had they all?

6. Hannah was at school 6 hours per day for 4 days, and 3 hours on the other day. How many hours was she at school that week?

7. Warren had 5 hens and each hen had 7 chickens. How many chickens were there?

8. A grocer sold 5 pounds of sugar at 5 cents a pound, and received in payment a fifty-cent piece. How much change should he give back.

9. Henry counted a company of soldiers in this way: he found that there were 8 rows of 9 men each and three officers. How many soldiers did he see?

10. Some children found 5 birds' nests containing 5 eggs each, and 4 nests containing 4 eggs each. How many eggs did they find?

11. A farmer sold 5 calves at \$5 each, and 5 sheep at \$4 each. How much did he receive for all?

12. Mary paid 10 cents for a dozen peaches, and 2 cents apiece for a dozen bananas. How much did they all cost her?

13. A boy worked 6 days for 5 dimes per day, but he spent 2 half-dimes for car fare each day. How much did he save?

14. Two brothers worked during the summer vacation of 10 weeks, one for \$6 per week and the other for \$3 per week. How much more did the one earn than the other in that time?

15. The pupils in a school were divided into 8 classes or grades, each containing 9 pupils. If every class contained 5 girls and 4 boys, how many boys were there in the school? How many girls? How many pupils?

16. A merchant employed 5 clerks at \$8 per week, and 2 at \$10 per week. How much did he pay them all per week?

17. The retail price of asparagus was 15 cents a bunch,

and the wholesale price was 12 cents. How much is a retail dealer's profit on 20 bunches which he bought at wholesale?

18. A hatter bought 12 hats at \$4 apiece, and sold them at \$6 apiece. How much did he gain?

19. A clothier sold overcoats at \$12.50 which cost him \$9.50. How much was his gain on 20 coats?

20. After paying \$2 apiece for repairing clocks that cost \$10 apiece, a man sold 12 of them for \$144. Did he gain or lose by the sale, and how much?

WRITTEN EXERCISES.

63. 1. A bookkeeper received \$75 per month for his services, but he spent in a year \$680. How much did he save in a year?

2. A train composed of 23 cars, containing 145 sheep each, arrived at Boston and the sheep were sold at \$5 a head. How much was received for them?

3. A boy had \$350 when he was 15 years old, and he saved \$100 per year until he was 21. How much had he then?

4. A manufacturer employed 118 workmen whose average wages per month were \$68. How much did they all earn per month?

5. A drover bought 83 cattle at \$37.25 per head. He sold 50 of them at \$45 per head, and the rest at \$35.25 per head. How much did he gain?

6. A tea merchant sold 348 pounds of tea at 57 cents per pound and gained \$13.55. How much did it cost him?

7. A and B each began business with \$4000. A gained \$600 the first year and B lost an equal sum. The second year A gained \$318 and B gained \$895. How much was each worth then?

8. A farmer raised 895 bushels of oats. He kept 50 bushels for seed, and enough to keep 10 horses, allowing 15 bushels for each horse, and sold the rest at 35 cents per bushel. How much did he receive for the amount sold?

9. A coal dealer sold 315 tons of coal at \$5.25 per ton. He received in payment a note for \$1000 and the rest in cash. How much cash did he receive?

10. The profits of a merchant were \$5896.75. He paid \$1391 for his household expenses, and 3 times as much for other expenses. What was his net gain?

11. What will be the cost of 485 chests of tea, each containing 55 pounds, at 38 cents a pound?

12. I had a farm of 313 acres. 185 acres yielded me an income of \$6.25 per acre, and the rest \$6 per acre. How much was my entire income from the farm?

13. My agent sold 3 lots of goods. For the first he received \$945, for the second twice as much as for the first, and for the third 3 times as much as for the other two. How much did he receive for all?

14. Three men formed a partnership. A contributed \$690, B twice as much, and C three times as much as A and B. What was their entire capital?

15. A drover bought 132 cattle at \$45 per head, and 67 at \$61 per head. He sold them all at \$50 per head. Did he gain or lose, and how much?

16. A farmer sold his farm at a gain of \$1685. The barns cost him \$1855, his house \$8520, and the land twice as much as the barns and house. How much did he get for the farm?

17. A merchant bought 35 pieces of carpet, each containing 49 yards, at \$1.18 per yard. He paid cash for a part of the cost and a note to settle the account for \$1250. How much cash did he pay?

18. Three men began business at the same time. During the first year A made a profit of \$855, B of 3 times as

much, and C of 4 times as much as A. At the beginning of the second year they formed a partnership in which each man contributed his gain of the previous year. What was the entire capital?

19. The daily sales of a newspaper for 5 days were 16,845 copies. If the papers were sold for 3 cents each, what was the amount received?

20. I know a man whose income is \$85.75 per day. How much does he receive in a year, or 365 days?

21. The annual expense of a school for 5 years was \$3825.25, exclusive of the wages of the teachers. In the school there were 11 teachers whose average annual salary was \$815.85. What was the annual expense of the school?

22. It was discovered in taking a census that a certain town contained 145 houses, that each house, on the average, contained 2 families, and that each family averaged 6 persons. How many people were there in the town?

23. A man bought a farm of 189 acres. He sold 99 acres at \$87 per acre, and thus reduced the cost of the rest to \$58 per acre. How much did the farm cost him?

24. A single poppy has been known to contain 32,000 seeds. How many seeds will 335 such poppies contain?

25. In a codfish 9,334,000 eggs have been found. If from each egg a fish was hatched, how many fishes were hatched from the eggs of 27 such codfish?

26. A man set out on foot to travel 600 miles. He traveled during the first 5 days 45 miles per day; during the next 6 days 38 miles per day. How many miles had he still to travel?

27. A ship left port, sailing at the rate of 9 miles per hour. After sailing for 23 hours she was driven directly back by a storm at the rate of 12 miles per hour for 10 hours. How far was she from the port from which she sailed, at the end of 30 hours?

DIVISION.

64. 1. How many groups of 2 squares can be formed from 6 squares? How many 2's are there in 6?

2. How many groups of 3 oranges can be formed from 9 oranges? How many 3's are there in 9?

3. Henry earns 6 cents an hour. In how many hours can he earn 42 cents? How many 6's are there in 42?

4. Louise arranged 56 spools in rows, placing 7 in each row. How many rows did she form?

5. Daisy's brother was ill for 49 days. How many weeks was he ill? How many 7's are there in 49?

6. Anna had 63 chickens which were hatched in broods of 9 chickens each. How many broods were there? How many 9's are there in 63?

7. A lad rides each day 8 miles in going to and from school. In how many days will he ride 96 miles? How many times is 8 contained in 96?

8. Mollie arranged 12 books in 4 piles. How many books were there in each pile?

9. How many apples will each child receive, when 10 apples are divided equally among 5 children?

10. What have you been doing with the above numbers?

65. The process of finding how many times one number is contained in another, or the process of separating a number into equal parts, is called **Division**.

66. The number to be divided is the **Dividend**.

67. The number by which we divide is the **Divisor**.

68. The result obtained by division is the **Quotient**.

69. The part of the dividend remaining when the division is not exact, is the **Remainder**.

70. The **Sign of Division** is \div . It is read *divided by*. When placed between two numbers, it shows that the one at the left is to be divided by the one at the right.

Thus, $24 \div 6$ is read, 24 divided by 6.

71. Division is also indicated by writing the dividend above the divisor, with a line between them.

Thus, $\begin{array}{r} 24 \\ 6 \end{array}$ is read, 24 divided by 6.

Another method of indicating division is by writing the divisor at the left of the dividend, with a line between them.

Thus, $6 \overline{)24}$ is read, 24 divided by 6.

DRILL EXERCISES.

72. Give the results instantly:

$9 \times 2 = ?$	$9 \times ? = 18$	$? \times 2 = 18$	$18 \div 2 = ?$	
$6 \times 7 = ?$	$6 \times ? = 42$	$? \times 7 = 42$	$42 \div 7 = ?$	
$5 \times 8 = ?$	$5 \times ? = 40$	$? \times 8 = 40$	$40 \div 8 = ?$	
$7 \times 5 = ?$	$7 \times ? = 35$	$? \times 5 = 35$	$35 \div 5 = ?$	
$8 \times 3 = ?$	$8 \times ? = 24$	$? \times 3 = 24$	$24 \div 3 = ?$	
$4 \times 9 = ?$	$4 \times ? = 36$	$? \times 9 = 36$	$36 \div 9 = ?$	
$6 \times 8 = ?$	$6 \times ? = 48$	$? \times 8 = 48$	$48 \div 8 = ?$	
$20 \div 10$	$21 \div 7$	$48 \div 12$	$50 \div 10$	$22 \div 2$
$8 \div 4$	$48 \div 4$	$24 \div 4$	$27 \div 9$	$32 \div 8$
$20 \div 2$	$42 \div 6$	$16 \div 2$	$66 \div 6$	$60 \div 5$
$96 \div 8$	$44 \div 11$	$24 \div 6$	$60 \div 12$	$88 \div 11$
$6 \div 3$	$28 \div 4$	$30 \div 5$	$63 \div 7$	$40 \div 5$
$22 \div 11$	$36 \div 4$	$66 \div 11$	$48 \div 8$	$44 \div 4$
$12 \div 2$	$70 \div 10$	$8 \div 2$	$49 \div 7$	$96 \div 12$
$20 \div 4$	$63 \div 9$	$81 \div 9$	$35 \div 7$	$24 \div 3$

24 ÷ 12	88 ÷ 8	35 ÷ 5	90 ÷ 10	45 ÷ 9
18 ÷ 3	55 ÷ 11	80 ÷ 10	48 ÷ 6	108 ÷ 9
60 ÷ 6	84 ÷ 7	10 ÷ 2	110 ÷ 11	121 ÷ 11
15 ÷ 3	16 ÷ 4	77 ÷ 11	80 ÷ 8	108 ÷ 12
56 ÷ 7	14 ÷ 7	6 ÷ 2	99 ÷ 9	21 ÷ 3
90 ÷ 9	4 ÷ 2	36 ÷ 9	40 ÷ 8	132 ÷ 12
70 ÷ 7	16 ÷ 8	84 ÷ 12	50 ÷ 5	120 ÷ 10
72 ÷ 8	72 ÷ 12	18 ÷ 2	27 ÷ 3	110 ÷ 10
20 ÷ 5	30 ÷ 3	54 ÷ 9	100 ÷ 10	120 ÷ 12
12 ÷ 6	18 ÷ 6	12 ÷ 3	24 ÷ 8	132 ÷ 11
33 ÷ 11	10 ÷ 5	12 ÷ 4	9 ÷ 3	144 ÷ 12
40 ÷ 4	15 ÷ 5	33 ÷ 3	36 ÷ 6	42 ÷ 7
25 ÷ 5	30 ÷ 10	32 ÷ 4	77 ÷ 7	60 ÷ 10
30 ÷ 6	24 ÷ 2	40 ÷ 10	99 ÷ 11	56 ÷ 8
18 ÷ 9	72 ÷ 6	64 ÷ 8	28 ÷ 7	36 ÷ 3
54 ÷ 6	36 ÷ 12	55 ÷ 5	72 ÷ 9	14 ÷ 2

73. When the divisor is not greater than 12.

ORAL EXERCISES.

1. When coal is selling at \$ 5 per ton, how many tons can be bought for \$ 45 ?

2. How many rows of trees are there in an orchard containing 60 trees arranged in rows containing 6 trees each ?

3. If a railroad ticket to Chicago costs \$ 7, how many such tickets can be bought for \$ 35 ?

4. A girl paid 96 cents for muslin at 8 cents per yard. How many yards did she buy ?

5. How many dictionaries, at \$ 8 apiece, can be bought for \$ 64 ?

6. How long will it take a boy to save \$ 48 if he saves \$ 4 per month ?

7. Howard paid \$ 9 for a suit of clothes. How many such suits can be bought for \$ 54 ?

8. How many gallons of molasses are there in 44 quarts ?

9. A peck measure holds 8 quarts of grain. How many pecks are there in 72 quarts?

10. If 8 nails are required to fasten a horseshoe, how many horseshoes can be fastened with 80 nails?

11. How many pairs of boots, at \$6 a pair, can be bought for \$48?

12. A farmer had 63 bales of hay. If he drew it away by drawing 7 bales at a load, how many loads did it make?

13. A grocer put 96 quarts of vinegar into two-gallon jugs. How many jugs did he use?

14. A stick measured 72 inches in length. How many feet long was it?

15. Henry found that he had added 66 figures, and that there were 6 figures in each column. How many columns did he add?

16. Mary knew that there were 72 desks in her school-room, and that there were 8 desks in a row, but she could not tell how many rows there were. Can you tell?

17. A grocer had 76 eggs. How many dozen did he have, and how many over?

18. A lad saw 48 birds sitting upon 6 telegraph wires, and he noticed that the same number sat upon each wire. How many were there in each group?

19. Lottie had saved 47 cents which she wished to divide equally, as far as possible, among four charitable societies. How much did she give each, and how much had she left?

20. How many times can a four-quart measure be filled from 49 quarts? How much will be left?

21. A postman found that he walked 24 miles per day in delivering mail. If he made 4 trips per day, how many miles did he walk each trip?

22. Susie spends $\frac{1}{4}$ of the year with her aunt and the rest of the year at home. How many months does she spend with her aunt?

WRITTEN EXERCISES.

74. 1. Divide 1702 by 3.

Divisor. Dividend.

$$\begin{array}{r} 3 \overline{)1702} \end{array}$$

$$567 - 1, \text{ Rem.}$$

Quotient.

EXPLANATION.—For convenience, the divisor is written at the left of the dividend, and the quotient below it or above it.

We begin at the left to divide. Thus, 3 is not contained in 1 thousand any thousand times, therefore the quotient cannot contain units of any order higher than hundreds. Hence, we find how many times 3 is contained in all the hundreds of the dividend.

1 thousand plus 7 hundreds are 17 hundreds ; 3 is contained in 17 hundreds 5 hundred times and 2 hundreds remainder. The 5 is written in hundreds' place in the quotient, and the remainder, 2 hundreds, is united with the tens.

2 hundreds plus 0 tens are 20 tens ; 3 is contained in 20 tens 6 times, and a remainder of 2 tens. The 6 is written in tens' place in the quotient, and the remainder, 2 tens, is united with the units.

2 tens plus 2 units are 22 units ; 3 is contained in 22 units 7 times, and a remainder of 1. The 7 is written in units' place in the quotient, and the remainder at a little distance to the right ; or it may be written as a part of the quotient by writing the divisor under it.

Hence, the quotient is 567 and 1 remainder, or $567\frac{1}{3}$.

PROOF.—567, the quotient, multiplied by 3, the divisor, plus 1, the remainder, is equal to 1702, the dividend. Hence, the work is correct.

75. When examples in division are solved without writing the products or remainders, the process is called **Short Division**.

Short division is not generally employed when the divisor exceeds 12.

Divide by short division, and prove :

2.	3.	4.	5.
$4 \overline{)3624}$	$6 \overline{)65814}$	$7 \overline{)59273}$	$8 \overline{)614653}$
906	10969	8467 $\frac{1}{4}$	76831 $\frac{1}{8}$
6. $4872 \div 4$.	11. $3936 \div 4$.	16. $\$1125 \div 9$.	
7. $6830 \div 5$.	12. $3234 \div 3$.	17. $\$1477 \div 7$.	
8. $2970 \div 5$.	13. $5256 \div 6$.	18. $\$4986 \div 9$.	
9. $2976 \div 6$.	14. $1362 \div 3$.	19. $\$6765 \div 5$.	
10. $4635 \div 3$.	15. $1328 \div 8$.	20. $\$3836 \div 7$.	

21. $9872 \div 6$.	28. $\$9185 \div 4$.	35. $\frac{1892}{8}$.
22. $4567 \div 5$.	29. $\$84364 \div 7$.	36. $\frac{1422}{8}$.
23. $\$89.32 \div 6$.	30. $\$38857 \div 8$.	37. $\frac{2787}{7}$.
24. $\$81.74 \div 9$.	31. $\$56878 \div 6$.	38. $\frac{1836}{4}$.
25. $\$83424 \div 8$.	32. $\$91341 \div 9$.	39. $\frac{52883}{4}$.
26. $\$73265 \div 5$.	33. $\$72144 \div 6$.	40. $\frac{53891}{8}$.
27. $\$58624 \div 7$.	34. $\$31486 \div 11$.	41. $\frac{52883}{12}$.

42. How many coats, at \$5 each, can be bought for \$2250?

43. If a man spends \$7 per week, how long will it take him to spend \$1477?

44. The salaries of 5 men amounted to \$3765. What was the average salary?

45. If a ship sails 8 miles per hour, how long will it take her to sail 1248 miles?

46. A stage traveled 6 miles per hour. How many hours would it require to travel 2352 miles?

47. At a church collection, each person contributed, on the average, 7 cents. If the entire sum collected was 4823 cents, how many persons contributed?

48. How many yards of cloth, at 9 cents per yard, can be bought for 29,439 cents?

49. How many cloaks, at \$5 each, can be bought for \$32,615?

50. In measuring the depth of the sea, 6 feet are called a fathom. How many fathoms deep is the sea when it measures 3516 feet?

Divide each of the following numbers by 3, 4, 5, 6, 7, 8, and 9.

51. 1950.	55. 5616.	59. 17286.	63. 277223.
52. 1476.	56. 6909.	60. 11615.	64. 548337.
53. 4360.	57. 2616.	61. 44854.	65. 389648.
54. 2943.	58. 1884.	62. 18809.	66. 261950.

76. When the divisor is expressed by more than one figure.

ORAL EXERCISES.

1. At 10 cents a yard, how many yards of cambric can be bought for 40 cents ?

2. There are 25 cents in a quarter of a dollar. To how many quarter dollars are 50 cents equal ?

3. If I pay \$ 60 for 20 lambs, how much do I pay for each ?

4. A real estate dealer divided 30 acres into 10 equal lots. How many acres were there in each ?

5. If a newsboy sells 40 papers per day, in how many days will he sell 120 papers ?

6. If I sell books at 30 cents each, how many must I sell to receive 90 cents ?

7. How many 10's are there in 60 ? In 90 ? In 120 ? In 250 ? In 350 ? In 480 ?

8. Since in 60 there are 6 tens; in 90, 9 tens; in 120, 12 tens; in 250, 25 tens, how may any number be divided by 10 ?

9. How many 100's are there in 600 ? In 900 ? In 1200 ? In 3600 ? In 7500 ? In 8400 ?

10. Since in 600 there are 6 hundreds; in 900, 9 hundreds; in 1200, 12 hundreds; in 3600, 36 hundreds, how may any number be divided by 100 ?

11. How many 1000's are there in 8000 ? In 25000 ?

12. Since in 8000 there are 8 thousands, in 25000 there are 25 thousands, how may a number be conveniently divided by 1000 ?

How may a number be divided by 10, 100, 1000, or by 1 with any number of ciphers annexed ?

77. *A number may be divided by 10, 100, 1000, or by 1 with any number of ciphers annexed, by cutting off from the right of the dividend as many figures as there are ciphers on the right of the divisor.*

WRITTEN EXERCISES.

78. Divide:

$$\begin{array}{r} 1. \ 1|0)3968|0 \\ \underline{3968} \end{array}$$

$$\begin{array}{r} 2. \ 1|00)382|00 \\ \underline{382} \end{array}$$

$$\begin{array}{r} 3. \ 1|00)468|57 \\ \underline{468} \ 57 \\ \underline{570} \end{array}$$

4. 3720 by 10.

9. 57386 by 100.

14. 52830 by 1000.

5. 4860 by 10.

10. 42570 by 100.

15. 32685 by 1000.

6. 3879 by 10.

11. 29830 by 100.

16. 97237 by 1000.

7. 4265 by 10.

12. 34206 by 100.

17. 54862 by 1000.

8. 3842 by 10.

13. 51309 by 100.

18. 69154 by 1000.

19. Divide 38476 by 500.

$5|00)384|76$
 $\underline{76} \ 476$

EXPLANATION. — The number is divided by 100 by cutting off the two right hand figures of the dividend. Dividing 384 by 5 is, therefore, dividing 38,400 by 500. The quotient is, therefore, 76 and 4 hundreds remainder. This remainder, added to the remainder obtained by dividing by 100, gives the entire remainder, 476; or the quotient is $76\frac{4}{5}$.

Divide:

20. 6835 by 20.

24. 98746 by 300.

28. 83426 by 500.

21. 7291 by 30.

25. 38295 by 400.

29. 31284 by 800.

22. 4863 by 60.

26. 41283 by 700.

30. 67395 by 700.

23. 3925 by 80.

27. 61579 by 900.

31. 81293 by 600.

32. Divide 15989 by 37.

Divisor. Dividend. Quotient.

$$\begin{array}{r} 37)15989(432\frac{5}{37} \\ \underline{148} \\ 118 \\ \underline{111} \\ 79 \\ \underline{74} \\ 5 \end{array}$$

EXPLANATION. — When the divisor is larger than 12 the steps in the solution are usually written. Thus, 37 is contained in 159 *four* times and a remainder of 11. The 4 is written in the quotient and the next figure of the dividend annexed to the remainder, making 118.

37 is contained in 118 *three* times and a remainder of 7. The 3 is written in the quotient and the next figure of the dividend

annexed to the remainder, making 79.

37 is contained in 79 *two* times and a remainder of 5. Therefore, the quotient is $432\frac{5}{37}$.

When the steps in the solution of an example are written, the process is called **Long Division**.

79. From the solution of the example given, the following are seen to be the successive steps taken in solving an example in Long Division :

Write the divisor at the left of the dividend, with a curved line between them.

Find how many times the divisor is contained in the fewest figures on the left hand of the dividend that will contain it, and write the quotient at the right of the dividend or above it. Multiply the divisor by this quotient, and place the product under the figures divided. Subtract the result from the partial dividend used, and to the remainder annex the next figure of the dividend.

Divide as before until all the figures of the dividend have been annexed to the remainder.

If any partial dividend will not contain the divisor, write a cipher in the quotient, then annex the next figure of the dividend, and proceed as before.

If there is a remainder after the last division, write it after the quotient, or with the divisor under it as part of the quotient.

PROOF. — *Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will equal the dividend.*

1. The quotient figure may generally be readily obtained by finding how many times the *first figure* of the divisor is contained in the *first figures* of the dividend that will contain it, *making allowance* for the addition to the product from the *second figure* of the divisor.

2. If the product of the divisor by the quotient figure is greater than the partial dividend from which it is to be subtracted, the quotient figure is *too large*.

3. Each remainder must be less than the divisor; otherwise the quotient figure is *too small*.

4. When there is no remainder, the divisor is said to be *exact*.

33.	34.	35.
83)51875(625	74)42683(576 $\frac{5}{2}$	52)45297($871\frac{5}{2}$
<u>498</u>	<u>370</u>	<u>416</u>
207	568	369
<u>166</u>	<u>518</u>	<u>364</u>
415	503	57
<u>415</u>	<u>444</u>	<u>52</u>
	59	5

Find the quotients of:

- | | | |
|----------------|----------------|----------------|
| 36. 4202 ÷ 11. | 40. 2310 ÷ 42. | 44. 2747 ÷ 41. |
| 37. 2556 ÷ 12. | 41. 3796 ÷ 52. | 45. 5084 ÷ 62. |
| 38. 1386 ÷ 21. | 42. 6840 ÷ 15. | 46. 3367 ÷ 91. |
| 39. 1488 ÷ 31. | 43. 1728 ÷ 12. | 47. 3024 ÷ 84. |

Divide each of the following numbers by 21, 32, 43, 54, 65:

- | | | | |
|------------|------------|------------|-------------|
| 48. 38425. | 53. 64136. | 58. 35794. | 63. 703821. |
| 49. 65484. | 54. 99874. | 59. 13570. | 64. 492532. |
| 50. 94653. | 55. 20562. | 60. 88992. | 65. 669046. |
| 51. 45677. | 56. 71117. | 61. 46823. | 66. 749638. |
| 52. 84732. | 57. 40505. | 62. 95895. | 67. 838327. |

Divide each of the following numbers by 37, 28, 19, 46, 35:

- | | | | |
|------------|------------|------------|-------------|
| 68. 37905. | 72. 29823. | 76. 19198. | 80. 469832. |
| 69. 35911. | 73. 50847. | 77. 34440. | 81. 269724. |
| 70. 98634. | 74. 72556. | 78. 66984. | 82. 123461. |
| 71. 48208. | 75. 96054. | 79. 85745. | 83. 777738. |

Divide:

- | | |
|------------------|----------------------|
| 84. 50854 by 94. | 86. 559108 by 202. |
| 85. 56394 by 78. | 87. 1547250 by 2134. |

DRILL TABLE.

	<i>A.</i>	<i>B.</i>	<i>C.</i>	<i>D.</i>	<i>E.</i>	<i>F.</i>	<i>G.</i>
1.	4	7	42	294	882	4410	9076352
2.	5	4	36	144	576	3456	4487063
3.	6	3	24	72	360	3240	6470342
4.	7	8	40	320	960	7680	1598788
5.	5	7	56	392	1568	15680	7421077
6.	2	8	72	576	1152	4608	6480713
7.	3	3	27	81	729	5103	2835108
8.	8	7	28	196	1960	9800	5306792
9.	4	8	64	512	1536	3072	4350787
10.	3	5	40	200	1800	5400	7863444
11.	6	6	30	180	7200	28800	6095162
12.	7	9	27	243	1701	8505	7453745
13.	9	8	48	384	768	2304	6666666
14.	3	6	36	216	864	3456	8580437
15.	8	8	56	448	2240	4480	4654380
16.	3	7	63	441	3528	7056	7820194
17.	9	7	49	343	3773	15092	9003068
18.	4	6	42	252	1008	5040	3905746
19.	8	9	54	486	2430	7290	2849370
20.	4	5	40	200	1000	9000	6470181
21.	9	9	36	324	972	8748	5783900
22.	6	8	24	192	768	3840	4300125
23.	5	9	81	729	1458	3916	1758374
24.	3	8	72	576	2304	9216	9999999
25.	5	6	54	324	1296	6480	9876543

EXERCISES UPON THE TABLE.

ORAL EXERCISES.

80. 1. $A \times B = ?$ 2. $C \div B = ?$ 3. $C - B = ?$

WRITTEN EXERCISES.

4. $C + D + E = ?$ 9. $D \times C = ?$ 14. $E \div B = ?$
 5. $C + D + E + F = ?$ 10. $C \times 10 = ?$ 15. $E \div C = ?$
 6. $E - D = ?$ 11. $D \times 400 = ?$ 16. $F \div C = ?$
 7. $F - E = ?$ 12. $D \times 203 = ?$ 17. $F \div D = ?$
 8. $D \times B = ?$ 13. $D \div B = ?$ 18. $100000 - F = ?$
 19. $G \div D = ?$ 21. $G - (D + E + F) = ?$
 20. $F \times D = ?$ 22. $A \times B \times C = ?$

REVIEW EXERCISES.**ORAL EXERCISES.**

81. 1. A bookseller bought 15 books for \$30 and sold them at a gain of \$15. How much apiece did he get for them?

2. A miller sold 8 barrels of flour at \$5 a barrel and bought with the proceeds cloth at \$4 per yard. How many yards did he purchase?

3. The cost of 10 tons of coal was \$50, but I sold it at a gain of \$20. How much did I receive per ton?

4. A bookseller bought 12 dictionaries at \$8 apiece and sold them at \$10 apiece. How much did he gain?

5. A steam yacht ran up a river for 8 hours at 10 miles per hour, and it came down in 5 hours. What was its rate per hour coming down?

6. The cost of 5 tons of coal at \$5 per ton was \$7 less than the cost of 8 cords of wood. What did the wood cost per cord?

7. The expense of sending delegates to a convention was \$65. If the entire expense had been \$1 less, the expense to each member would have been \$2. How many members were there?

8. A man owed a debt amounting to \$84. If he paid \$60 of the sum in 5 months, in how long a time will he pay it all at that rate?

9. A farmer sold a grocer 12 pounds of butter at 20¢ a pound and took in exchange brooms at 24 cents apiece. How many brooms did he get?

10. Two boys set out from the same place and traveled in the same direction, Arthur traveling 5 miles per hour and Bennie 3 miles per hour. How soon will Arthur be 18 miles ahead of Bennie?

11. How far has each traveled when Arthur is 24 miles ahead?

12. Suppose they travel in opposite directions at those rates, how far apart will they be in 8 hours ?

13. In how many days can 4 men do as much work as 8 men can do in 6 days ?

14. Howard's father gave him an acre of land to till so that he might earn his own pocket money. He sold the products of the land for \$ 38, but he paid \$ $5\frac{1}{2}$ for seeds to plant and \$ $2\frac{1}{2}$ for help. How much money had he left ?

15. A boy bought 6 lead-pencils for 24¢ and exchanged them for some erasers at 3¢ each. How many erasers did he get ?

16. A man bought a carriage for \$ 120. After spending \$ 40 in repairing and painting it, he sold it for \$ 150. Did he gain or lose, and how much ?

17. A newsboy bought 20 papers for 30 cents and sold them at a gain of 30 cents. For how much apiece did he sell them ?

18. Henry bought 6 car-tickets for 25 cents. How many could he buy for 50 cents ?

19. A clerk saved \$ 20 a month. In how many months could he save enough to buy a library worth \$ 240 ?

20. A jeweler bought 12 watches for \$ 72 and sold them at a gain of \$ 3 each. At what price did he sell each of them ?

21. The wages of a machinist were \$ 21 per week, and his expenses were \$ 10. In how many weeks can he pay a debt of \$ 75 and deposit \$ 46 in a savings bank ?

WRITTEN EXERCISES.

82. 1. If a man paid \$ 7564 for 124 horses, how much did each cost ?

2. At \$ 9 a yard, how many yards of velvet can I buy for \$ 207 ?

3. How many weeks are there in 1736 days ?

4. If 12 horses cost \$ 1560, what will one horse cost ?
What will 5 horses cost at the same rate ?
5. How many dozen eggs are there in 1728 eggs ?
6. Divide \$ 800.25 equally among 15 men.
7. In an orchard there are 25 rows of trees. If there are 3200 trees in all, how many trees are there in a row ?
8. How many times 26 bushels are there in 22,568 bushels ?
9. How many acres of land will \$ 22,575 buy, at \$ 75 an acre ?
10. In 64 loads there were 143,360 pounds. How many pounds were there in a load ?
11. How many days are there in 3600 hours ? In as many minutes ?
12. How many hours are there in 4320 minutes ? How many days ?
13. What number, multiplied by 23, will give 36,087 ?
14. Product, 62,397 ; multiplier, 27 ; find the multiplicand.
15. If the product of two factors is 21,015, and one factor is 45, what is the other factor ?
16. If you weigh 1476 ounces, how many pounds (of 16 ounces) do you weigh ? How many pounds does your sister weigh, her weight being 133 ounces less than yours ?
17. If the receipts of a railroad for one year (52 weeks) are \$ 2,683,928, how much are they per week ?
18. The combined wealth of 405 men is \$ 1,247,400. How much would each have if the money were equally divided among them ?
19. How many schooners, each carrying 8700 bushels of wheat, will be required to carry 843,900 bushels ?
20. Suppose that two cans of equal size together hold 376 pints. How many gallons are there in each ?
21. How many poor families may be supplied from 37

barrels of flour, allowing 28 pounds to each, a barrel of flour weighing 196 pounds ?

22. How many thousands make one million ?

23. If a railroad company pays \$ 1750 for constructing a car, how many cars can be made for \$ 75,250 ?

24. The average daily receipts of a ferry-boat are \$ 325. In how many days will the receipts amount to \$ 76,050 ?

25. The stock of a railroad company is \$ 2,565,000. If it is divided into shares of \$ 500 each, how many shares are there ?

26. Mount Everest, in Asia, is said to be 29,100 feet high. Since there are 5280 feet in a mile, how many miles high is it ?

27. In what number is 224 contained 28 times ?

28. Mr. Smith earns \$ 28 while Mr. Allen earns \$ 15. How much will Mr. Smith earn while Mr. Allen earns \$ 120 ?

29. How many times will a cart-wheel, 15 feet in circumference, revolve in going a mile, or 5280 feet ?

30. How many posts and how many rails will be required for a fence 312 feet long, if the posts are set 12 feet apart, and the fence is 4 rails high ?

31. From a farm containing 1000 acres, there were sold at one time 274 acres ; at another time, 238 acres. How many acres remained unsold ?

32. Divide \$ 432 between Mr. A. and Mr. B., so that Mr. B. will have \$ 16 more than Mr. A.

33. At an election, the sum of the votes received by two opposing candidates was 6742. If the successful candidate received 328 more votes than his opponent, how many votes did each receive ?

34. Into a tank were poured 76 barrels of kerosene, of 43 gallons each. If 2489 gallons are drawn from the tank, how many gallons are left ?

35. If I buy 8 boxes of starch, each box containing 24 pounds, at 8 cents a pound, and offer in payment a \$ 20 bill, what change shall I receive ?

36. Elsie and Lottie have together \$5.70, and Lottie has 40 cents more than Elsie. How much money has each ?

37. If 14 men can do a piece of work in 56 days, how long will it take 28 men to do half the work ?

38. I bought two horses. The first cost \$312, and the second \$40 less than the first. How much did the two horses cost ?

39. If 19 tons of coal run an engine 703 miles, how far will 17 tons run it ?

40. A farmer exchanged 4 cows, worth \$79 each, for a span of horses. What were the horses worth apiece ?

41. A horse cost \$235, a carriage \$219, and a hack 3 times as much as both. What did all three cost ?

42. A merchant bought 42 bales of cloth, each bale containing 36 pieces, and each piece 38 yards, at 9 cents a yard. He sold the whole at 11 cents a yard. How much did he gain ?

43. If I take 764 bushels from 2000 bushels twice, how much will remain ?

44. A drover with \$2144 bought as many horses as possible for \$165 each, and spent the remainder for sheep at \$4 a head. How many of each did he buy ?

45. How many times 945 will make 23,625 ?

46. Twenty men built a school-house in 56 days. In how many days could 70 men have built it ?

47. A man bought 163 barrels of flour at \$9 a barrel; 15 barrels were spoiled, and the remainder he sold at \$11 a barrel. Did he gain or lose, and how much ?

48. A school uses 36 crayons a day. Since a gross is 12 dozen, how many gross will it use in 40 weeks, provided there is no school on Saturday ?

49. If it takes 5 yards of cloth to make a pair of shirts, what will 24 pairs cost at 15 cents per yard for the cloth, 45 cents apiece for bosoms, wristbands, and buttons, and 75 cents apiece for making?

50. A clerk has a salary of \$42 a month, and his expenses are \$27 a month. How many years will it take him to lay up \$900?

51. Find the cost of 267 yards of cotton at 13 cents a yard, and twice as much at 12 cents a yard.

52. There are in a certain school 37 pupils 14 years old; 76 pupils 12 years old; 114, 11 years old; 149, 10 years old; and 168, 9 years old. What is their average age?

53. A man paid \$375 for a piano, \$13 for freight and cartage, and \$2 for tuning it. He then rented it for 7 quarters at \$15 a quarter, and afterwards sold it for \$325. Did he gain or lose, and how much?

54. Two men leave the same place and travel in opposite directions, one at the rate of 30 miles a day, and the other at the rate of 37 miles per day. How far will they be apart at the end of 14 days?

55. If the men traveled in the same direction, how far apart would they be in the same time?

56. A, B, and C have together \$198. A and C have \$145. A and B have \$99. How much has each?

57. Howard, Roy, and Jamie went fishing, and together caught 77 fish. Of these, Howard and Jamie together caught 58. Howard's and Roy's, when put together, numbered 54. How many did each catch?

58. A farmer sold 18 calves at the rate of 3 for \$33. How much did he get for them?

59. If 117 tons of coal are worth \$702, how much are 235 tons worth?

60. If 12 barrels of flour are worth \$132, what will 36 barrels cost?

61. If a farmer receives \$2275 for 13 horses, for how much would 27 horses sell at the same rate?

62. A's farm contains 227 acres; B's lacks 12 acres of being twice as much; C's lacks 42 acres of being 3 times as much as B's; and D's contains as much as A's and C's. How many acres are there in each farm?

63. A farmer wished to obtain \$260. He sold 72 bushels of wheat at \$1.50 per bushel, and enough apples at \$2 per barrel to obtain the sum required. How many barrels of apples did he sell?

64. If a bookkeeper receives \$1400 per year for his services, and his expenses are \$840, in what time can he save enough to buy 32 acres of land at \$140 per acre?

65. A man starting out on a journey took \$200. He paid for railroad fare \$67; for berth in sleeping car, 4 days, \$2 a day; for hotel bills, 15 days, \$3 a day; and for other expenses, \$25. How much money had he left?

66. The greater of two numbers is 4056, and their difference is 3650. What is the less number?

67. The divisor is 534, and the quotient 43. What is the dividend?

68. The subtrahend is 34,203, and the remainder is 8706. What is the minuend?

83. The parenthesis, (), shows that the numbers included within it are to be subjected to the same operation.

Thus, $(15 - 3) \times 3$ means that $15 - 3$, or 12, is to be multiplied by 3; but $15 - 3 \times 3$ means that 15 is to be diminished by 3×3 , or 9.

$(7 + 2) \times 5$ means that the sum of 7 and 2 is to be multiplied by 5; but $7 + 2 \times 5$ means that 7 is to be increased by 2×5 .

$(7 + 5) \div 6$, or $\frac{7+5}{6}$, means that the sum of 7 and 5 is to be divided by 6.

$[(9 + 3) \times 2 - 3] \div 7$ means that the sum of 9 and 3 is to be multiplied by 2, the product diminished by 3, and the remainder divided by 7.

$(7 \times 8) - (5 \times 5)$ means that the product of 7 and 8 is to be diminished by the product of 5 and 5.

84. The Vinculum, —, or brackets [], may be used instead of the parenthesis.

Thus, $\overline{5 - 3} \times 3$ or $[5 - 3] \times 3$ may be used instead of $(5 - 3) \times 3$.

85. In finding the value of expressions in which the parenthesis, vinculum, or brackets are used,

First, perform the operations on the numbers that are written within parentheses and brackets, or under the vinculum, as indicated by the signs. Next, multiply and divide, as indicated by the signs \times and \div . Finally, add and subtract, as indicated by the signs $+$ and $-$.

Find the values of the following:

1. $(4 + 7) \times 5$.
2. $4 + 7 \times 5$.
3. $(8 - 3) \times 2$.
4. $8 - 3 \times 2$.
5. $4 + 16 \div 4$.
6. $(4 + 16) \div 4$.
7. $(3 + 4) \times (8 - 5)$.
8. $\frac{7 + 5}{3} + \frac{7 - 3}{4}$.
9. $2 \times 12 \div 2 \times 3$.
10. $2 \times 12 \div (2 \times 3)$.
11. $18 - \frac{(3 \times 4) - (2 \times 3)}{3}$.
12. $[(5 + 6) \times 4 - 5 \times 8] \times 9$.
13. $(3 \times 4) \times 5 - (9 + 7) \div 8$.
14. $2 + 12 \div 4 + (8 + 8 \div 4) \div 2$.
15. $(312 + 36) - (381 - 215) - 65$.
16. $(214 - 81) - (115 - 18 + 6) + 10$.
17. $(413 - 200) - (118 - \overline{24 - 4} + 6) + 3$.
18. $(171 - 86) - (\overline{3 \times 4} + 27) + 10$.
19. $(3 \times 4) \times 9 - (4 + 12) \div 4 + 5$.
20. $(5 + 2 + 6) \times 4 - (7 + 8 - 3) \div 6 + 10$.
21. $\frac{(36 - 7) \times 5 + (102 + 6) \div 9}{}$.
22. $\overline{(99 - 3) + 8 - (86 + 10) + 12} + \overline{(3 + 6) \div 3}$.

FACTORING.

- 86.** 1. What is the product when 3 and 2 are multiplied together? What are 3 and 2 of their product? (Art. 86.)
2. What factors will produce 9?
3. What numbers when multiplied together will produce 12? What are 3 and 4, or 6 and 2 of their product?
4. What are the factors of 20? Of 36? Of 15?
5. What are the factors of 27? Of 25? Of 32?
6. What are the factors of 21? Of 33? Of 50?
7. What are the factors of 63? What else may 7 be called of 63 besides a factor?
8. If 9 is one of two factors of 18, what is the other factor? If 3 is one of the factors? If 6 is one of the factors?
9. What numbers will exactly divide 18? 25? 36?
10. Give the exact divisors of 42; 96; 35; 50; 27; 72.
11. Give the factors of 36; 40; 48; 70; 80.
12. Give the exact divisors of 44; 56; 64; 84; 96.
13. Name the exact divisors of 49; 88; 63; 24; 27.
14. What numbers between 0 and 10 cannot be exactly divided by any number except themselves and 1? What numbers between 10 and 20? Between 20 and 30?
15. What numbers between 0 and 10 can be exactly divided by other numbers besides themselves and 1? Between 10 and 20? Between 20 and 30?
16. Select from the following the numbers that have no exact divisors except themselves and 1: 35, 42, 63, 56, 61, 47, 49, 81, 37, 26, 18, 45.

17. Select from the following the numbers that have exact divisors besides themselves and 1: 24, 36, 41, 39, 27, 45, 33, 37, 50, 44, 60, 71, 72.

87. A number that expresses whole units is called an **Integer**.

Thus, 5, 27, 35 are integers, or integral numbers.

88. The integers which, upon being multiplied together will produce the number, are called **Factors** of the number.

Thus, 5 and 3 are the factors of 15.

89. An integer which will divide a number without a remainder is called an **Exact Divisor** of the number.

Thus, 2, 3, 6, and 9 are exact divisors of 18. They are also *factors* of 18.

90. A number that has no exact divisors except itself and 1 is called a **Prime Number**.

Thus, 1, 3, 5, 7 are prime numbers.

91. A number that has exact divisors besides itself and 1 is called a **Composite Number**.

Thus, 24, 36, 40, 100 are composite numbers.

92. Factors that are prime numbers are **Prime Factors**.

Thus, 7 and 5 are the prime factors of 35.

93. A number that is exactly divisible by 2 is called an **Even Number**.

Thus, 8, 12, 20, 24 are even numbers.

94. A number that is not exactly divisible by 2 is called an **Odd Number**.

Thus, 15, 21, 35, 43 are odd numbers.

95. The process of separating a number into its factors is called **Factoring**.

TESTS OF DIVISIBILITY.

96. 1. Make a list of numbers from 1 to 126, which have 2 for one or more of their factors or divisors. Notice what the right-hand, or units' figure, of each is.

2. Make a list of numbers from 1 to 100 which have 5 for one or more of their factors or divisors. Notice what the units' figure of each is.

3. Make a list of numbers from 1 to 100 which have 3 for one or more of their factors or divisors. Divide the sum of the digits of each of these numbers by 3, and notice the remainder, if any.

4. Make a list of numbers from 1 to 200 which have 9 for one or more of their factors or divisors. Divide the sum of the digits of each by 9, and notice the remainder, if any

97. It is apparent, therefore, that:

A number is divisible by

2 if the units' figure is 2, 4, 6, 8, or 0.

5 if the units' figure is 0 or 5.

3 if the sum of its digits is divisible by 3.

9 if the sum of its digits is divisible by 9.

98. Tell by inspection which of the numbers, on page 129, columns *C* and *D*, are divisible by 2; by 5; by 3; by 9.

Which in column *E* are divisible by 2; by 3; by 5; by 9.

WRITTEN EXERCISES.

99. 1. What are the prime factors of 336?

$$\begin{array}{r}
 2 \overline{) 336} \\
 2 \overline{) 168} \\
 2 \overline{) 84} \\
 2 \overline{) 42} \\
 3 \overline{) 21} \\
 7
 \end{array}$$

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7.$$

itself, we continue the process, and obtain the prime factors of 336.

EXPLANATION.— Since every factor of a number is a divisor of it, we may find the *prime* factors of 336 by dividing by the *exact* divisors that are *prime* numbers. Dividing by 2, we find the factors of 336 to be 2 and 168. But 168 has a factor 2, and, since a factor of a factor of a number is a factor of the number

What are the prime factors

- | | | |
|--------------|---------------|---------------|
| 2. Of 168 ? | 12. Of 625 ? | 22. Of 1280 ? |
| 3. Of 224 ? | 13. Of 912 ? | 23. Of 2625 ? |
| 4. Of 144 ? | 14. Of 832 ? | 24. Of 3204 ? |
| 5. Of 198 ? | 15. Of 990 ? | 25. Of 1155 ? |
| 6. Of 216 ? | 16. Of 1008 ? | 26. Of 2500 ? |
| 7. Of 484 ? | 17. Of 435 ? | 27. Of 1485 ? |
| 8. Of 576 ? | 18. Of 222 ? | 28. Of 1296 ? |
| 9. Of 432 ? | 19. Of 672 ? | 29. Of 2016 ? |
| 10. Of 672 ? | 20. Of 775 ? | 30. Of 2875 ? |
| 11. Of 396 ? | 21. Of 289 ? | 31. Of 2890 ? |

CANCELLATION.

ORAL EXERCISES.

100. 1. How many times is 2 times 5 contained in 4 times 5 ? 2 times 3 in 4 times 3 ? 2 times any number in 4 times that number ?

2. How many times is 4 times 7 contained in 8 times 7 ? 4 times 35 in 8 times 35 ? 4 times a certain number in 8 times the same number ?

3. How many times is 6×12 contained in 18×12 ? 5×23 in 15×23 ? 7×47 in 21×47 ?

4. What is the quotient of $(24 \times 17) \div (12 \times 17)$? Of $(63 \times 24) \div (9 \times 24)$? Of $(48 \times 61) \div (24 \times 61)$? Of $(36 \text{ times } 19) \div (18 \text{ times } 19)$?

5. In determining the quotient, what numbers may be omitted from both dividend and divisor ?

101. From the solution of the examples given, it is evident that :

Rejecting equal factors from both dividend and divisor does not alter the quotient.

WRITTEN EXERCISES.

102. 1. Divide $4 \times 6 \times 8 \times 10$ by $2 \times 3 \times 4 \times 15$.

$$\begin{array}{cccc} 2 & 2 & 2 & 2 \\ 4 & 6 & 8 & 10 \\ \hline 2 & 3 & 4 & 15 \\ 1 & 1 & 1 & 3 \end{array} = \frac{16}{3} = 5\frac{1}{3}.$$

EXPLANATION.—The dividend is written above the divisor, with a line between them as in division.

Since the factors 2, 3, and 4 are factors of 4, 6, and 8 in the dividend, we reject them from both dividend and divisor, leaving the factors 2, 2, and 2 in the dividend. Since the factor 5 is common to 10 in the dividend and to 15 in the divisor, it is also rejected, leaving 2 in the dividend, and 3 in the divisor.

The product of the uncanceled factors of the dividend is 16, and of the divisor 3. Hence, the quotient is $\frac{16}{3}$, or $5\frac{1}{3}$.

2. Divide $7 \times 4 \times 11 \times 8$ by $22 \times 4 \times 4 \times 21$.

$$\begin{array}{cccc} & & & 1 \\ 1 & 1 & 1 & 2 \\ 7 & 4 & 11 & 8 \\ \hline 22 & 4 & 4 & 21 \\ 2 & 1 & 1 & 3 \\ 1 & & & \end{array} = \frac{1}{3}.$$

EXPLANATION.—In this example, the common factor 7 is rejected from 7 and 21, leaving 1 in the dividend and 3 in the divisor.

Next, the common factor 4 is removed from 4 in the dividend and 4 in the divisor. Then the common factor 4 is removed from 8 in the dividend and 4 in the divisor, leaving 2 in the dividend and 1 in the divisor. Then the common factor 11 is rejected from 11 in the dividend and from 22 in the divisor, leaving 1 in the dividend and 2 in the divisor. Then the common factor 2 is removed from the dividend and the divisor.

The product of $1 \times 1 \times 1 \times 1$ in the dividend is 1.

The product of $1 \times 1 \times 1 \times 3$ in the divisor is 3.

Hence, the quotient is $1 \div 3$, or $\frac{1}{3}$.

1. When the factor is contained once in any number, the 1 may be omitted; but it is better to write it until the process is clearly understood.

2. When all the factors of both divisor and dividend are canceled, the quotient is 1, for the dividend will then exactly contain the divisor *once*.

Find the value of:

3. $\frac{4 \times 5 \times 7 \times 9}{2 \times 2 \times 6 \times 7 \times 3}.$

4. $\frac{8 \times 9 \times 12 \times 16}{4 \times 3 \times 5 \times 6 \times 20}.$

$$5. \frac{2 \times 3 \times 8 \times 12 \times 24}{6 \times 4 \times 36 \times 4}$$

$$7. \frac{15 \times 20 \times 25 \times 27}{10 \times 15 \times 18 \times 25}$$

$$6. \frac{18 \times 24 \times 32 \times 36}{9 \times 48 \times 4 \times 18}$$

$$8. \frac{25 \times 30 \times 40 \times 35}{20 \times 15 \times 18 \times 7}$$

Divide, using cancellation:

$$9. 40 \times 48 \times 54 \times 60 \text{ by } 30 \times 24 \times 72 \times 3.$$

$$10. 30 \times 49 \times 64 \times 25 \text{ by } 15 \times 35 \times 24 \times 10.$$

$$11. 12 \times 60 \times 36 \times 70 \text{ by } 28 \times 48 \times 6 \times 5.$$

$$12. 32 \times 36 \times 33 \times 45 \text{ by } 24 \times 30 \times 44 \times 9.$$

$$13. 30 \times 36 \times 24 \times 42 \text{ by } 45 \times 27 \times 8 \times 28.$$

$$14. 27 \times 32 \times 45 \times 36 \text{ by } 18 \times 24 \times 9 \times 6.$$

$$15. 45 \times 28 \times 36 \times 56 \text{ by } 49 \times 4 \times 18 \times 18.$$

$$16. 27 \times 28 \times 35 \times 49 \text{ by } 18 \times 21 \times 40 \times 28.$$

$$17. 45 \times 60 \times 63 \times 28 \text{ by } 27 \times 20 \times 25 \times 21.$$

$$18. 55 \times 36 \times 27 \times 42 \text{ by } 12 \times 25 \times 35 \times 33.$$

$$19. 36 \times 64 \times 25 \times 40 \text{ by } 32 \times 50 \times 18 \times 10.$$

$$20. 56 \times 18 \times 32 \times 49 \text{ by } 16 \times 36 \times 42 \times 28.$$

$$21. 32 \times 45 \times 72 \times 15 \text{ by } 64 \times 18 \times 36 \times 30.$$

$$22. 64 \times 15 \times 63 \times 24 \text{ by } 30 \times 21 \times 56 \times 12.$$

$$23. 44 \times 20 \times 50 \times 36 \text{ by } 55 \times 10 \times 25 \times 18.$$

$$24. 40 \times 48 \times 21 \times 18 \text{ by } 56 \times 36 \times 35 \times 24.$$

$$25. 40 \times 27 \times 32 \times 3 \text{ by } 5 \times 8 \times 12 \times 8 \times 27.$$

26. A lady bought 9 yards of ribbon at 56 cents per yard, but exchanged it for other ribbon at 32 cents per yard. How many yards did she get?

27. At the rate of \$129 for 27 acres of land, what will 180 acres cost?

28. At what price per yard will 5 bales of cloth, containing 12 pieces of 42 yards each, pay for 50 rolls of carpeting, of 75 yards each, at \$2.10 per yard?

29. A merchant purchased 13 firkins of butter, each containing 39 pounds, at 32 cents a pound, and paid for it with

4 patterns of silk of 13 yards each. How much was the silk worth per yard?

30. A grocer exchanged 27 chests of tea, each containing 48 pounds, worth 40 cents a pound, for 18 pieces of dress goods, each containing 60 yards. What was the price per yard of the dress goods?

31. A stationer had 28 packages of cards, each containing 24 cards, which he exchanged for 21 packages of note paper, each containing 24 sheets, worth 2 cents per sheet. What were the cards worth apiece?

32. Four farms, each containing 80 acres, worth \$72 per acre, were exchanged for 5 farms, each containing 96 acres. What was the value per acre of the farms received in exchange?

33. A farmer exchanged 8 bushels of wheat, at \$2 per bushel, for 3 pieces of muslin, worth 10 cents per yard. How many yards were there in each piece?

$$34. \frac{5 \times 8 \times 7 \times 12 \times 16 \times 24 \times 10 \times 9}{3 \times 16 \times 18 \times 4 \times 20 \times 6 \times 5 \times 8} = ?$$

$$35. \frac{9 \times 8 \times 11 \times 16 \times 15 \times 35 \times 42 \times 18}{6 \times 21 \times 7 \times 5 \times 25 \times 32 \times 4 \times 9} = ?$$

$$36. \frac{12 \times 9 \times 30 \times 25 \times 16 \times 36 \times 48}{18 \times 6 \times 15 \times 35 \times 72 \times 24 \times 9} = ?$$

$$37. \frac{42 \times 35 \times 18 \times 24 \times 60 \times 64 \times 72}{36 \times 32 \times 12 \times 25 \times 30 \times 21 \times 5} = ?$$

$$38. \frac{44 \times 20 \times 36 \times 54 \times 65 \times 27 \times 48}{24 \times 40 \times 27 \times 72 \times 13 \times 45 \times 72} = ?$$

$$39. \frac{45 \times 60 \times 63 \times 28 \times 35 \times 24 \times 50}{25 \times 21 \times 56 \times 21 \times 40 \times 48 \times 36} = ?$$

$$40. \frac{36 \times 48 \times 21 \times 35 \times 42 \times 96 \times 56}{28 \times 84 \times 42 \times 24 \times 70 \times 14 \times 9} = ?$$

FRACTIONS.



- 103.** 1. Into how many parts are these squares divided ?
2. How do the parts compare as to size ?
3. What is each part called ?
4. What is a half of anything ?
5. How many halves are equal to a whole one ?
6. How many halves are there in 2 apples ? In 3 pears ?
In 5 peaches ? In 1 orange and a half ? In 2 lemons and a half ? In 4 and a half plums ?
7. Two half melons are equal to how many whole melons ? 4 half melons ? 6 half melons ? 3 half melons are equal to how many whole melons, and how many half melons besides ? 5 half melons ?
8. Draw a line. Divide it into halves. Then divide each half into two equal parts.
9. Into how many parts is the whole line divided ?
10. What is each part called ?
11. What do you call 2 of the parts ? 3 of the parts ?
How many thirds are equal to a whole thing ?
12. How many fourths are there in any whole thing ?
13. How many fourths are there in 2 circles ? In 3 squares ? In 1 line and 1 fourth of a line ? In 2 lines and 3 fourths of a line ?

14. How many fourths of an apple are there in half of an apple?

15. What part of a half is a fourth?

16. Which is the greater, 1 half of a yard or 2 fourths of a yard? 1 pineapple or 3 fourths of a pineapple?

17. How many whole apples are equal to 8 fourths of an apple? To 12 fourths? How many whole apples and how many fourths besides are equal to 5 fourths? To 7 fourths? To 9 fourths? To 10 fourths?

18. Draw a circle. Divide it into fourths. Then divide each fourth into 2 equal parts.

19. Into how many equal parts is the circle divided?

20. What is each part called?

21. What do you call 2 of the parts? 3 of the parts? 5 of the parts? 6 of the parts? 7 of the parts?

22. How many eighths are there in a pie? In 2 pies? In 2 pies and 1 eighth of a pie? In 3 pies and 3 eighths of a pie? In 1 fourth of a pie? In 3 fourths of a pie? In 1 pie and 3 fourths of a pie? In half a pie? In 2 and a half pies?

23. How many whole sticks of candy are equal to 8 eighths? 16 eighths? How many whole sticks of candy and how many eighths besides are equal to 10 eighths? To 15 eighths? To 17 eighths? To 20 eighths?

————— 24. What is each part of this line
called?

25. What do you call 2 of these parts?

26. How many thirds are there in the whole of anything?

27. How many thirds are there in 3 lines? In 5 lines? In 2 lines and 1 third of a line? In 4 and 2 thirds lines?

28. If I have 6 thirds of orange, to how many whole oranges is that equal? 9 thirds? 12 thirds?

————— 29. How is this line divided?

30. What is each part called? What

do you call 2 parts? What other name will express the same part of the line that 2 sixths expresses?

31. What do you call 3 parts of this line? What other name will express the part of the line that 3 sixths expresses?

32. What do you call 4 parts of this line? What other name will express the part of the line that 4 sixths expresses?

33. How many sixths are there in 2 sheets of paper? 3 sheets? 4 sheets? 5 sheets?

34. Which is the greater, 1 third of a pie, or 1 sixth of a pie?

35. What part of a third is a sixth?

36. How many sixths are there in two thirds? In 1 peach and 1 third of a peach? In 2 and a half peaches? In 3 and 5 sixths peaches?

37. How many units and how many sixths besides are there in 7 sixths? 9 sixths? 12 sixths? 14 sixths?

38. Draw a line. Divide it into sixths. Now divide each sixth into 2 equal parts.

39. Into how many equal parts is the line divided?

40. What is each part called?

41. How many twelfths are there in the whole of anything?

42. How many twelfths of a line are equal to a sixth? To a third? To a half? To 2 thirds? To 5 sixths? To 1 and 5 sixths? To 2 and a half? To 2 and 2 thirds?

43. How many units and how many twelfths besides are equal to 15 twelfths? To 18 twelfths? To 20 twelfths?

44. What part of a sixth is a twelfth? What other name will express the part of the line that 2 twelfths expresses? 4 twelfths? 6 twelfths? 8 twelfths? 10 twelfths?

45. Draw three parallel lines of just the same length.

46. Divide the first one into halves, the second one into fourths, and the third one into eighths.

47. Draw three other parallel lines of just the same length as the first group.

48. Divide the first one into thirds, the second into sixths, and the third into twelfths.

49. Which is the greater, 1 half or 1 third ?

50. Which is the greater, 1 half or 2 thirds ?

51. When you divide the halves into 2 equal parts, what parts of the whole do you have ?

52. When you divide the thirds into 2 equal parts, what parts of the whole do you have ?

53. One third is equal to how many sixths ?

54. Two thirds are equal to how many sixths ?

55. Three thirds are equal to how many sixths ?

56. What is 1 half of 1 half ? 1 half of 1 third ?

57. Which is the greater, 1 third or 1 fourth ?

58. Which is the greater, 1 fourth or 1 sixth ?

59. Which is the greater, 1 third or 2 sixths ?

60. What parts of the whole do you get by dividing the fourths into 2 equal parts ?

61. One fourth is equal to how many eighths ?

62. Two fourths are equal to how many eighths ?

63. Three fourths are equal to how many eighths ?

64. Which is the greater, 1 sixth or 1 eighth ?

65. How many sixths are equal to a half ?

66. How many eighths are equal to a half ?

67. Which is the greater, 3 sixths or 3 eighths ?

68. What parts of the whole do you get by dividing the sixths into 2 equal parts ?

69. One sixth is equal to how many twelfths ?

70. Two sixths are equal to how many twelfths ?

71. Three sixths are equal to how many twelfths ? Four sixths ? 5 sixths ? 6 sixths ?

72. Which is the greater, 1 twelfth or 1 eighth ?
73. How many twelfths are equal to a half ?
74. How many eighths are equal to a half ?
75. Which is the greater, 6 twelfths or 6 eighths ?
76. When anything is divided into 5 equal parts, what is each part called ?

77. What do you call 3 of the parts ?
78. How many fifths are there in the whole of anything ?
79. Draw a line and divide it into fifths. Divide each fifth into 2 equal parts. Into how many equal parts is the line divided ?

80. What is each part called ?
81. How many tenths are there in the whole of anything ?
82. How many tenths are equal to a fifth ?
83. Draw a line and divide it into thirds. Divide each third into 3 equal parts. Into how many equal parts have you divided the line ? What is each part called ?
84. How many ninths are equal to a third of the line ?

104. One or more of the equal parts of anything is a **Fraction**.

105. Two numbers written one above the other with a line between them are used to express a fraction.

106. The number which shows into how many equal parts the unit has been divided is called the **Denominator**. It is written below the line.

Thus, in the fraction $\frac{3}{5}$, 5 is the denominator. It shows that the unit has been divided into five equal parts.

107. The number which shows how many parts form the fraction is called the **Numerator**. It is written above the line.

Thus, in the fraction $\frac{3}{5}$, 3 is the numerator. It shows that the fraction contains 3 of the 5 equal parts into which the unit was divided.

108. The numerator and denominator are called the **Terms of a Fraction.**

109. A fraction in which the numerator is less than the denominator is called a **Proper Fraction.**

Thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ are proper fractions.

The value of a proper fraction is, therefore, less than 1.

110. A fraction in which the numerator equals or exceeds the denominator is called an **Improper Fraction.**

Thus, $\frac{5}{4}$, $\frac{7}{3}$, $\frac{9}{2}$ are improper fractions.

The value of an improper fraction is, therefore, 1 or more than 1.

111. A number expressed by an integer and a fraction is a **Mixed Number.**

Thus, $6\frac{1}{2}$, $8\frac{3}{4}$, $6\frac{1}{3}$ are mixed numbers.

112. A fraction may be regarded as expressing *unexecuted division*.

Thus, $\frac{15}{4}$ is equal to $15 \div 4$; $\frac{24}{6}$ is equal to $24 \div 6$.

113. 1. Read and interpret the expression $\frac{5}{13}$.

EXPLANATION. — $\frac{5}{13}$ represents 5 of the 13 equal parts into which the unit has been divided. It also represents one thirteenth of 5, and 5 divided by 13. It is read *five thirteenths*.

Read and interpret the following expressions :

- | | | | | |
|--------------------|--------------------|----------------------|-----------------------|----------------------|
| 2. $\frac{3}{8}$. | 5. $\frac{2}{7}$. | 8. $\frac{7}{10}$. | 11. $\frac{11}{20}$. | 14. $\frac{23}{8}$. |
| 3. $\frac{5}{8}$. | 6. $\frac{3}{8}$. | 9. $\frac{5}{12}$. | 12. $\frac{10}{11}$. | 15. $\frac{3}{4}$. |
| 4. $\frac{7}{9}$. | 7. $\frac{3}{4}$. | 10. $\frac{8}{15}$. | 13. $\frac{17}{24}$. | 16. $\frac{10}{8}$. |

Express by figures:

17. Five sevenths. Ten fourteenths. Seven twelfths.
18. Eight ninths. Nine tenths. Five thirteenths.
19. Nineteen twenty-fifths. Thirty-eight fortieths.
20. Thirty thirty-ninths. Forty-one sixty-eighths.
21. Forty-six eightieths. Twenty-seven forty-fifths.
22. Five two-hundred-sixths. Seven three-hundred-ninths.
23. One eighth of 17. One thirty-fifth of 59.
24. Fifteen divided by 17. Twenty-four divided by 83.

REDUCTION OF FRACTIONS.

114. To reduce integers or mixed numbers to fractions.

1. A man received \$2 for a day's work. If he was paid in half dollars, how many did he get?
2. How many quarter dollars are equal to \$2?
3. How many sixths are there in 2 pies? Eighths?
4. In three things how many halves are there? Fourths? Eighths? Thirds? Sixths? Twelfths? Fifths? Tenths?
5. How many fourths are there in 2? In $2\frac{1}{4}$? In $2\frac{3}{4}$?
6. How many eighths are there in 1? In $3\frac{3}{8}$? In $3\frac{7}{8}$?
7. How many sixths are there in 3? In $1\frac{5}{6}$? In $2\frac{5}{6}$?
8. How many fifths are there in 2? In $2\frac{3}{5}$? In $3\frac{4}{5}$?
9. How many sevenths are there in 1? In $1\frac{4}{7}$? In $4\frac{2}{7}$?

115. The process of changing the form of fractions without changing their value is called **Reduction of Fractions.**

WRITTEN EXERCISES.

116. 1. Reduce $12\frac{3}{8}$ to eighths.

$12 = \frac{96}{8}$. **EXPLANATION.** — Since in 1 there are 8 eighths, in 12 there are 12 times 8 eighths, or $\frac{96}{8}$; and in $12 + \frac{3}{8}$ there are $\frac{96}{8} + \frac{3}{8}$, or $\frac{99}{8}$. Hence, $12\frac{3}{8}$ is equal to $\frac{99}{8}$.

117. From the above solution it is seen that:

In changing a mixed number to a fraction, the integer is multiplied by the given denominator, to this product the numerator of the fractional part is added, and the result is written over the given denominator.

Reduce the following to improper fractions :

- | | | | |
|----------------------|------------------------|------------------------|------------------------|
| 2. $6\frac{5}{11}$. | 7. $13\frac{3}{4}$. | 12. $36\frac{5}{18}$. | 17. $53\frac{1}{2}$. |
| 3. $9\frac{3}{8}$. | 8. $18\frac{5}{9}$. | 13. $44\frac{7}{22}$. | 18. $65\frac{1}{16}$. |
| 4. $7\frac{5}{9}$. | 9. $21\frac{6}{11}$. | 14. $35\frac{8}{28}$. | 19. $74\frac{3}{4}$. |
| 5. $8\frac{7}{12}$. | 10. $32\frac{7}{10}$. | 15. $37\frac{3}{5}$. | 20. $87\frac{3}{11}$. |
| 6. $9\frac{3}{10}$. | 11. $18\frac{6}{11}$. | 16. $24\frac{1}{10}$. | 21. $98\frac{1}{4}$. |

118. To reduce improper fractions to integers or mixed numbers.

1. To how many oranges are 4 half oranges equal? 6 half oranges? 8 half oranges? 5 half oranges?

2. To how many dollars are 4 quarter dollars equal? 8 quarter dollars? 16 quarter dollars? 5 quarter dollars?

3. In 10 half bushels how many bushels are there? In 12 half bushels? In 20 half bushels?

4. How many units are there in 3 thirds? In 6 thirds? In 12 thirds? In 15 thirds? In 7 thirds?

5. How many units are there in $\frac{8}{8}$? $\frac{8}{8}$? $\frac{12}{8}$? $\frac{14}{8}$? $\frac{18}{8}$?

6. How many yards are there in $\frac{8}{8}$ yards? $\frac{18}{8}$ yards? $\frac{12}{8}$ yards? $\frac{16}{8}$ yards? $\frac{25}{8}$ yards?

7. How many units are there in $\frac{8}{8}$? $\frac{8}{8}$? $\frac{10}{8}$? $\frac{12}{8}$? $\frac{14}{8}$? $\frac{17}{8}$? $\frac{19}{8}$? $\frac{23}{8}$? $\frac{18}{8}$? $\frac{26}{8}$? $\frac{33}{8}$? $\frac{31}{8}$? $\frac{43}{8}$? $\frac{47}{8}$?

WRITTEN EXERCISES.

119. 1. Reduce $17\frac{5}{7}$ to a mixed number.

$17\frac{5}{7} = 125 \div 7 = 17\frac{5}{7}$. **EXPLANATION.**—Since 7 sevenths are equal to 1 unit, 125 sevenths are equal to as many units as 7 sevenths are contained times in 125 sevenths, or 17 $\frac{5}{7}$ units. Therefore, $17\frac{5}{7} = 17\frac{5}{7}$.

120. From the above solution, it is seen that:

In reducing an improper fraction to a mixed number the numerator is divided by the denominator.

Reduce to integers or mixed numbers.

- | | | | |
|-----------------------|------------------------|------------------------|-------------------------|
| 2. $\frac{158}{9}$. | 7. $\frac{294}{18}$. | 12. $\frac{605}{48}$. | 17. $\frac{963}{62}$. |
| 3. $\frac{266}{8}$. | 8. $\frac{365}{22}$. | 13. $\frac{899}{47}$. | 18. $\frac{897}{86}$. |
| 4. $\frac{395}{8}$. | 9. $\frac{439}{84}$. | 14. $\frac{999}{46}$. | 19. $\frac{2896}{59}$. |
| 5. $\frac{187}{4}$. | 10. $\frac{604}{27}$. | 15. $\frac{285}{74}$. | 20. $\frac{4986}{88}$. |
| 6. $\frac{528}{10}$. | 11. $\frac{899}{44}$. | 16. $\frac{845}{81}$. | 21. $\frac{8947}{96}$. |

121. To reduce fractions to smaller or larger terms.

1. In $\frac{1}{2}$ of a yard how many fourths of a yard are there? How many sixths? Eighths? Tenths?

2. In $\frac{1}{3}$ of an orange how many sixths are there? How many ninths? Twelfths?

3. How are the terms of the fraction $\frac{2}{4}$ obtained from those of $\frac{1}{2}$? $\frac{3}{6}$ from $\frac{1}{3}$?

4. Since the terms of the fraction $\frac{2}{4}$ are obtained from those of $\frac{1}{2}$ by multiplying them by 2, how may the terms of $\frac{1}{2}$ be obtained from those of $\frac{2}{4}$?

5. Since the terms of the fraction $\frac{3}{6}$ are obtained from those of $\frac{1}{2}$ by multiplying them by 3, how may the terms of $\frac{1}{2}$ be obtained from those of $\frac{3}{6}$?

122. From these examples, it is seen that:

The value of a fraction is not changed by multiplying or dividing both terms by the same number.

6. Change to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{6}$.

7. Change to twentieths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{7}{10}$; $\frac{9}{10}$.

8. Change $\frac{2}{3}$ to halves; $\frac{2}{3}$ to fourths; $\frac{4}{12}$ to thirds.

9. Change to thirds: $\frac{2}{3}$; $\frac{4}{6}$; $\frac{3}{6}$; $\frac{5}{6}$; $\frac{8}{12}$; $\frac{10}{12}$; $\frac{11}{12}$; $\frac{14}{12}$; $\frac{20}{12}$.

10. Change to fifths: $\frac{2}{10}$; $\frac{4}{10}$; $\frac{6}{10}$; $\frac{8}{10}$; $\frac{9}{15}$; $\frac{10}{15}$; $\frac{21}{15}$; $\frac{24}{15}$.

123. A number that is an exact divisor of two or more numbers is a **Common Divisor** of those numbers.

124. A fraction is expressed in its **Smallest** or **Lowest Terms** when its numerator and denominator have no **common divisor**.

WRITTEN EXERCISES.**125.** 1. Reduce $\frac{48}{64}$ to its smallest terms.

$\begin{array}{r} 8 \overline{)48} = \frac{6}{8} \\ 2 \overline{)64} = \frac{8}{8} \\ 2 \overline{)6} = \frac{3}{4} \end{array}$ **EXPLANATION.**— Since the fraction is to be reduced to its smallest terms, we divide the terms of the fraction by 8, and the terms of the resulting fraction by 2. The terms of the fraction $\frac{3}{4}$ have no common divisor; therefore $\frac{48}{64}$, when reduced to its smallest terms, is equal to $\frac{3}{4}$.

126. It is evident, therefore, that:

In reducing a fraction to its smallest terms, the numerator and denominator are divided by any common divisor, and this process is continued until the terms have no common divisor.

Reduce to their smallest terms:

2. $\frac{18}{88}$.	8. $\frac{44}{66}$.	14. $\frac{44}{100}$.	20. $\frac{54}{144}$.
3. $\frac{15}{25}$.	9. $\frac{24}{40}$.	15. $\frac{17}{70}$.	21. $\frac{160}{420}$.
4. $\frac{14}{22}$.	10. $\frac{32}{60}$.	16. $\frac{84}{280}$.	22. $\frac{150}{375}$.
5. $\frac{20}{30}$.	11. $\frac{18}{88}$.	17. $\frac{160}{288}$.	23. $\frac{326}{482}$.
6. $\frac{12}{22}$.	12. $\frac{15}{35}$.	18. $\frac{60}{240}$.	24. $\frac{350}{490}$.
7. $\frac{21}{38}$.	13. $\frac{55}{75}$.	19. $\frac{25}{125}$.	25. $\frac{875}{1050}$.

127. To reduce dissimilar to similar fractions.

1. If I bought $\frac{1}{2}$ of a pound of one kind of tea, and $\frac{1}{3}$ of a pound of another kind, how many sixths of a pound of each kind did I buy?

2. Mr. Roe planted $\frac{1}{2}$ of an acre with potatoes, and $\frac{1}{3}$ of an acre with corn. How many tenths of an acre did he plant with each?

3. Mr. Lee has $\frac{1}{3}$ of a barrel of russet apples and $\frac{1}{4}$ of a barrel of Baldwin apples. How many twelfths of a barrel has he of each?

4. Change $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having their denominators alike. $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{1}{2}$ and $\frac{1}{4}$. $\frac{1}{4}$ and $\frac{1}{5}$. $\frac{1}{4}$ and $\frac{1}{6}$. $\frac{1}{6}$ and $\frac{1}{8}$. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{10}$. $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{12}$. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$.

128. Fractions that have the same denominators are called **Similar Fractions**.

129. Fractions that have not the same denominators are called **Dissimilar Fractions**.

130. The denominator of similar fractions is called a **Common Denominator**.

131. When similar fractions are expressed in their lowest terms, they have their **Least Common Denominator**.

132. 1. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{12}$ to similar fractions.

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$$\frac{11}{12} = \frac{11 \times 2}{12 \times 2} = \frac{22}{24}$$

EXPLANATION. — Since the fractions are to be changed to other fractions having a common denominator, the terms of each fraction must be multiplied by some number which will cause them to have the same denominator.

By examining the denominators, 4, 8, and 12, it is evident that the denominators of all the fractions can be made 24, and the fractions will then be similar. To make the denominators 24, the terms of the first fraction must be multiplied by 6, the terms of the second by 3, the terms of the third by 2. And thus the fractions are changed to the similar fractions, $\frac{18}{24}$, $\frac{15}{24}$, $\frac{22}{24}$.

Reduce to similar fractions:

2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{12}$.

6. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{10}$.

10. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$.

3. $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{18}$.

7. $\frac{3}{8}$, $\frac{2}{10}$, $\frac{1}{5}$.

11. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$.

4. $\frac{5}{7}$, $\frac{2}{14}$, $\frac{1}{21}$.

8. $\frac{5}{8}$, $\frac{6}{12}$, $\frac{1}{4}$.

12. $\frac{1}{8}$, $\frac{1}{12}$, $\frac{2}{24}$, $\frac{5}{48}$.

5. $\frac{4}{9}$, $\frac{2}{18}$, $\frac{5}{36}$.

9. $\frac{1}{7}$, $\frac{2}{21}$, $\frac{1}{42}$.

13. $\frac{3}{4}$, $\frac{5}{11}$, $\frac{2}{22}$, $\frac{1}{11}$.

14. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{12}$, and $\frac{9}{16}$ to similar fractions having their least common denominator.

$$\begin{array}{r|rrrr} 3 & 3 & 4 & 12 & 16 \\ 2 & 1 & 4 & 4 & 16 \\ 2 & 1 & 2 & 2 & 8 \\ \hline & 1 & 1 & 1 & 4 \end{array}$$

$$3 \times 2 \times 2 \times 4 = 48.$$

EXPLANATION. — The least common denominator cannot always be easily found by inspection. It may then be found as in the margin.

Since the least common denominator must be the smallest number that will contain each of the denominators, it must contain each of the prime factors of the denominators and *no other factors*. The prime factors are found as in the margin. 3 is a prime factor of 3 and 12, and consequently a factor of the least common denominator. Dividing by 3, and writing below, the quotients and numbers of which 3 is not a factor, we have 1, 4, 4, 16. Dividing by 2, and again by 2, the factors of the denominator are found to be the divisors 3, 2, 2, and the factor 4 in the last row. Their product is 48, the least common denominator. The fractions thus become $\frac{32}{48}$, $\frac{36}{48}$, $\frac{28}{48}$, $\frac{27}{48}$.

133. From these examples it is evident that:

In reducing dissimilar to similar fractions the least common denominator is found by multiplying together all the different prime factors of the given denominators. The terms of each fraction are then multiplied by such a number as will cause each fraction to have the least common denominator.

1. In finding the product of the different prime factors, each factor must be used the greatest number of times that it occurs in any of the given numbers.

2. Reduce all mixed numbers to improper fractions.

Reduce to similar fractions having the least common denominator :

15. $\frac{3}{4}, \frac{7}{12}, \frac{9}{20}$.	20. $\frac{1}{8}, \frac{2}{7}, \frac{5}{6}$.	25. $1\frac{1}{2}, 1\frac{1}{3}, 4\frac{3}{8}$.
16. $\frac{5}{6}, \frac{6}{7}, \frac{9}{16}$.	21. $\frac{4}{5}, \frac{3}{8}, \frac{7}{15}$.	26. $4\frac{1}{7}, \frac{3}{8}, 2\frac{3}{4}$.
17. $\frac{3}{8}, \frac{5}{12}, \frac{7}{10}$.	22. $\frac{5}{11}, \frac{4}{5}, \frac{7}{14}$.	27. $6\frac{1}{2}, 1\frac{1}{4}, 1\frac{1}{8}$.
18. $\frac{4}{5}, \frac{3}{4}, \frac{4}{7}$.	23. $\frac{6}{5}, \frac{4}{12}, \frac{7}{80}$.	28. $5\frac{2}{3}, 6\frac{2}{3}, \frac{2}{8}$.
19. $\frac{5}{8}, \frac{3}{5}, \frac{7}{15}$.	24. $\frac{4}{9}, \frac{7}{36}, \frac{5}{6}$.	29. $3\frac{1}{3}, 4\frac{2}{3}, 1\frac{1}{6}$.

ADDITION OF FRACTIONS.

134. 1. John walked $\frac{1}{4}$ of a mile on Monday, $\frac{2}{8}$ of a mile on Tuesday, and $\frac{3}{8}$ of a mile on Wednesday. How many eighths of a mile did he walk in all ?

2. How many sixths are $\frac{2}{3}, \frac{3}{8},$ and $\frac{1}{4}$?

3. How many ninths are $\frac{4}{9}, \frac{1}{3},$ and $\frac{2}{9}$?

4. How many twelfths are $\frac{3}{12}, \frac{5}{12},$ and $\frac{1}{12}$?

5. $\frac{4}{7} + \frac{2}{7} = ?$ $\frac{2}{5} + \frac{1}{5} = ?$ $\frac{3}{10} + \frac{6}{10} = ?$ $\frac{5}{11} + \frac{3}{11} = ?$ $\frac{2}{4} + \frac{3}{4} = ?$
 $\frac{5}{6} + \frac{3}{6} = ?$ $\frac{5}{18} + \frac{7}{18} = ?$ $\frac{1}{30} + \frac{8}{30} = ?$

6. A boy spent $\frac{1}{4}$ of a dollar for a book, and $\frac{1}{2}$ of a dollar for a cap. What part of a dollar did both cost ?

7. A farmer sold $\frac{1}{3}$ of his sheep at one time and $\frac{1}{2}$ of them at another. What part of his sheep did he sell ?

8. How many sixths are there in $\frac{1}{3}$ and $\frac{1}{6}$?

9. How many ninths are there in $\frac{1}{3}$ and $\frac{1}{9}$?

10. How many eighths are there in $\frac{1}{4}$ and $\frac{1}{8}$?
11. How many tenths are there in $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{10}$?
12. What kind of fractions can be added without changing their form?
13. What must be done to dissimilar fractions before they can be added?
14. Add $\frac{3}{4}$ and $\frac{1}{2}$; $\frac{1}{2}$ and $\frac{3}{8}$; $\frac{1}{4}$ and $\frac{3}{8}$; $\frac{1}{8}$ and $\frac{5}{8}$; $\frac{2}{3}$ and $\frac{1}{6}$; $\frac{1}{3}$ and $\frac{4}{5}$; $\frac{1}{4}$ and $\frac{5}{12}$; $\frac{1}{8}$ and $\frac{7}{12}$; $\frac{2}{3}$ and $\frac{1}{12}$.
15. Add $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{5}$; $\frac{1}{5}$ and $\frac{1}{6}$; $\frac{1}{6}$ and $\frac{1}{4}$.
16. Add $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{1}{2}$ and $\frac{3}{8}$; $\frac{2}{3}$ and $\frac{3}{4}$; $\frac{3}{4}$ and $\frac{5}{6}$; $\frac{5}{6}$ and $\frac{4}{5}$.
17. I paid \$ $\frac{1}{2}$ for milk and \$ $\frac{1}{10}$ for lettuce. How much did I pay for both?
18. James paid \$ $\frac{3}{4}$ for a fishing-rod and \$ $\frac{1}{10}$ for a line. What part of a dollar did both cost him?
19. Lucy worked $\frac{1}{2}$ of a day at her lessons and spent $\frac{1}{4}$ of a day at a picnic. What part of the day did she spend at both?
20. A farmer's son sold $\frac{1}{2}$ dozen eggs to one man, $\frac{3}{4}$ dozen to another, and $\frac{2}{3}$ dozen to another. How many dozen did he sell in all?
21. A lad was presented with some money on his birthday. The next day he spent $\frac{1}{3}$ of it, the day following, $\frac{1}{4}$ of it, and on the third day he spent $\frac{1}{10}$ of it. What part of the money did he spend?

WRITTEN EXERCISES.

135. 1. What is the sum of $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{4}{5}$?

$$\frac{5}{8} = \frac{25}{40}$$

$$\frac{3}{4} = \frac{30}{40}$$

$$\frac{4}{5} = \frac{32}{40}$$

$$\frac{87}{40} = 2\frac{7}{40}$$

EXPLANATION. — Since the fractions are not similar, they must be changed to similar fractions before they are added.

The least common denominator of the given fractions is 40, and $\frac{5}{8} = \frac{25}{40}$, $\frac{3}{4} = \frac{30}{40}$, and $\frac{4}{5} = \frac{32}{40}$. Hence the sum of the given fractions will be the sum of $\frac{25}{40}$, $\frac{30}{40}$, and $\frac{32}{40}$, which is $\frac{87}{40}$, or $2\frac{7}{40}$.

136. From the above solutions, it is evident that:

In adding fractions, dissimilar fractions are first changed to similar ones, then their numerators are added, and the sum is placed over the common denominator.

1. If the sum is an improper fraction, it should be reduced to an integer or mixed number.

2. When there are mixed numbers or integers to be added, the integers and fractions should be added separately and then those results added.

Find the sum of the following :

2. $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{1}{4}$.

3. $\frac{3}{10}$, $\frac{7}{15}$, and $\frac{5}{12}$.

4. $\frac{5}{8}$, $\frac{4}{5}$, and $\frac{7}{15}$.

5. $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{1}{6}$.

6. $\frac{5}{16}$, $\frac{3}{20}$, and $\frac{4}{5}$.

7. $\frac{3}{7}$, $\frac{5}{28}$, and $\frac{3}{4}$.

8. $\frac{4}{7}$, $\frac{2}{21}$, and $\frac{2}{3}$.

9. $\frac{3}{20}$, $\frac{7}{12}$, and $\frac{5}{8}$.

10. $6\frac{2}{3}$, $2\frac{2}{3}$, and $4\frac{4}{15}$.

11. $3\frac{4}{7}$, $1\frac{8}{21}$, and $5\frac{1}{2}$.

12. $4\frac{2}{3}$, $3\frac{3}{8}$, and $8\frac{7}{18}$.

13. $5\frac{4}{5}$, $10\frac{1}{10}$, and $1\frac{1}{6}$.

14. $6\frac{2}{3}$, $4\frac{4}{5}$, and $1\frac{5}{12}$.

15. $12\frac{7}{8}$, $8\frac{2}{3}$, and $2\frac{3}{8}$.

16. $9\frac{4}{15}$, $\frac{7}{3}$, and $11\frac{1}{2}$.

17. $7\frac{7}{10}$, $3\frac{2}{5}$, and $6\frac{1}{10}$.

18. A farmer sold a load of hay for $\$8\frac{3}{4}$, a load of oats for $\$22\frac{1}{2}$, and a load of wheat for $\$53\frac{1}{4}$. How much did he receive for all?

19. James picked $5\frac{3}{8}$ bushels of apples from one tree, $7\frac{1}{2}$ bushels from another, $6\frac{3}{10}$ bushels from another, and $8\frac{5}{8}$ bushels from another. How many bushels did he pick in all?

20. Three barrels of sugar contained respectively $225\frac{3}{8}$ pounds, $232\frac{5}{12}$ pounds, and $240\frac{1}{12}$ pounds. What was the weight of the whole?

21. A school-room is $32\frac{2}{3}$ feet long and $29\frac{1}{3}$ feet wide. What is the distance around the room?

22. An express train starts from Boston at $1\frac{3}{4}$ P.M., reaches Springfield $2\frac{2}{3}$ hours later, and Albany $4\frac{3}{10}$ hours later still. At what o'clock does it arrive in Albany?

SUBTRACTION OF FRACTIONS.

137. 1. Bertie earned $\$ \frac{4}{5}$ and spent $\$ \frac{2}{5}$. How much had he left?

2. Susie had $\frac{5}{8}$ of a pound of candy, but she ate $\frac{3}{8}$ of a pound. How much had she left?

3. There were $\frac{5}{8}$ of a bushel of oats in a bin. If $\frac{2}{8}$ of a bushel was taken out, how much remained?

4. $\frac{5}{7} - \frac{2}{7} = ?$ $\frac{7}{12} - \frac{1}{12} = ?$ $\frac{5}{8} - \frac{3}{8} = ?$ $\frac{9}{10} - \frac{3}{10} = ?$
 $\frac{11}{16} - \frac{5}{16} = ?$ $\frac{13}{18} - \frac{8}{18} = ?$ $\frac{12}{25} - \frac{10}{25} = ?$

5. If I have $\$ \frac{3}{8}$ and give away $\$ \frac{1}{4}$, what part of a dollar shall I have left?

6. A man who had $\frac{7}{8}$ of a cord of wood sold $\frac{1}{2}$ of a cord. What part of a cord had he left?

7. One pitcher held $\frac{7}{8}$ of a quart of milk, and another held $\frac{3}{4}$ of a quart. How much more did the one hold than the other?

8. $\frac{1}{2} - \frac{1}{4} = ?$ $\frac{3}{4} - \frac{1}{2} = ?$ $\frac{1}{2} - \frac{1}{6} = ?$ $\frac{5}{6} - \frac{1}{2} = ?$ $\frac{1}{2} - \frac{1}{8} = ?$
 $\frac{7}{8} - \frac{1}{2} = ?$

9. What kind of fractions can be subtracted without changing their form?

10. What must be done to dissimilar fractions before they can be subtracted?

11. $1 - \frac{2}{3} = ?$ $1 - \frac{4}{5} = ?$ $1 - \frac{3}{4} = ?$ $1 - \frac{3}{8} = ?$ $1 - \frac{7}{9} = ?$

12. $\frac{1}{4} - \frac{1}{8} = ?$ $\frac{1}{3} - \frac{1}{6} = ?$ $\frac{1}{3} - \frac{2}{9} = ?$ $\frac{3}{4} - \frac{3}{8} = ?$ $\frac{2}{3} - \frac{5}{12} = ?$
 $\frac{11}{12} - \frac{5}{6} = ?$

13. $\frac{1}{2} - \frac{1}{3} = ?$ $\frac{1}{2} - \frac{2}{5} = ?$ $\frac{2}{3} - \frac{1}{4} = ?$ $\frac{4}{5} - \frac{2}{3} = ?$ $\frac{5}{6} - \frac{1}{4} = ?$
 $\frac{7}{8} - \frac{5}{12} = ?$

14. A lad earned one day $\$ \frac{3}{4}$ and spent $\$ \frac{3}{8}$. How much had he left?

15. A mechanic's wages were \$3 per day, but he paid $\$ \frac{3}{4}$ per day for his board. How much did he save daily?

16. A boy agreed to work for a man 5 days, but lost $1\frac{1}{4}$ days by sickness. How many days did he work?

17. A boy raised vegetables which he sold for \$3. The cost of seed and other expenses were \$1 $\frac{1}{10}$. How much was his net profit?

18. A man bought a hat for \$2 $\frac{1}{2}$, a vest for \$2 $\frac{3}{4}$, and a linen coat for \$1 $\frac{1}{2}$. How much change should he receive from the merchant, if he paid for them with a ten-dollar bill?

WRITTEN EXERCISES.

138. 1. What is the difference between $\frac{1}{8}$ and $\frac{7}{12}$?

EXPLANATION. — Since the fractions are not similar, they must be made similar before they can be subtracted. The least common denominator of the given fractions is 48; $\frac{1}{8} = \frac{6}{48}$, $\frac{7}{12} = \frac{28}{48}$. Hence, the difference between the given fractions is the difference between $\frac{6}{48}$ and $\frac{28}{48}$, which is $\frac{17}{48}$.

2. What is the difference between $15\frac{3}{4}$ and $8\frac{5}{6}$?

EXPLANATION. — Since the numbers are composed of integers and fractions, each may be subtracted separately. Reducing the fractions to similar fractions, it is evident that $\frac{1}{4}$ cannot be subtracted from $\frac{5}{6}$, hence 1, taken from 15, is united with $\frac{3}{4}$. 1, or $\frac{1}{1} + \frac{3}{4} = \frac{7}{4}$. Subtracting $8\frac{5}{6}$ from $14\frac{7}{4}$, there is a remainder of $6\frac{1}{12}$.

139. From the solution of the above examples it is seen that:

In subtracting fractions, dissimilar fractions are changed to similar ones, then their numerators are subtracted, and the difference is written over the common denominator.

When there are integers or mixed numbers to be subtracted, the integers and fractions should be subtracted separately.

	3.	4.	5.	6.	7.	8.	9.
Out of	$\frac{1}{16}$	$\frac{5}{7}$	$\frac{5}{16}$	$\frac{4}{16}$	$\frac{10}{16}$	$\frac{11}{16}$	$\frac{17}{16}$
Take	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{24}$	$\frac{7}{24}$	$\frac{2}{16}$	$\frac{6}{16}$	$\frac{11}{16}$

	10.	11.	12.	13.	14.	15.	16.
From	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{17}{14}$	9	20	15	26
Take	$\frac{2}{70}$	$\frac{2}{18}$	$\frac{11}{80}$	$2\frac{5}{11}$	$7\frac{2}{40}$	$10\frac{9}{18}$	$7\frac{11}{14}$
	17.	18.	19.	20.	21.	22.	23.
From	$6\frac{3}{8}$	$8\frac{5}{18}$	$11\frac{1}{8}$	$16\frac{3}{7}$	$20\frac{5}{14}$	$31\frac{9}{18}$	$63\frac{4}{18}$
Take	$2\frac{3}{7}$	$2\frac{5}{24}$	$5\frac{3}{8}$	$10\frac{5}{9}$	$16\frac{1}{16}$	$20\frac{1}{18}$	$39\frac{1}{11}$

24. A box of soap weighed $75\frac{3}{4}$ pounds. The weight of the box alone was $3\frac{5}{12}$ pounds. How much did the soap weigh?

25. From a farm of 120 acres there were sold $36\frac{1}{16}$ acres. How much was left?

26. A flag-staff $50\frac{2}{10}$ feet high was broken off by a storm, so that it measured $43\frac{7}{8}$ feet. How much was broken off?

27. A grocer had $128\frac{1}{4}$ pounds of sugar in one barrel, and $98\frac{3}{4}$ pounds in another. How much more was in the one barrel than the other?

28. A clerk earned \$75 per month. His expenses during that time were as follows: board, \$ $22\frac{1}{2}$; washing, \$ $5\frac{3}{8}$; and other expenses \$ $16\frac{3}{8}$. How much did he save per month?

29. A man spent $\frac{2}{10}$ of his money for a house, $\frac{1}{12}$ for furniture, and $\frac{3}{8}$ for horses and carriages. What part of his money had he left?

30. The sum of two numbers is $11\frac{5}{8}$. One of the numbers is $8\frac{1}{4}$. What is the other number?

31. From a web of cotton cloth containing $58\frac{7}{16}$ yards, $8\frac{1}{8}$ yards were cut off at one time, and $15\frac{3}{8}$ yards at another time. How many yards remained?

32. Mr. Allen left $\frac{3}{8}$ of his property to his wife, $\frac{2}{16}$ to his sons, $\frac{1}{4}$ to his daughter, and the remainder to charitable institutions. What part of his property was left to charitable institutions?

Find the value of the following :

33. $\frac{5}{8} + \frac{3}{12} + \frac{1}{24} + \frac{5}{16}$.

40. $3\frac{1}{2} + 4\frac{2}{3} + 3\frac{3}{4} - 1\frac{1}{4}$.

34. $\frac{3}{4} + \frac{5}{6} + \frac{7}{12} + \frac{1}{18}$.

41. $6\frac{3}{5} - 4\frac{2}{3} + 3\frac{1}{10} - 1\frac{8}{10}$.

35. $\frac{4}{5} + \frac{1}{10} - \frac{1}{4} - \frac{1}{8}$.

42. $8\frac{5}{9} + 2\frac{7}{18} - 3\frac{1}{3} - 3\frac{4}{9}$.

36. $\frac{6}{15} + \frac{3}{30} - \frac{1}{20} - \frac{1}{60}$.

43. $2\frac{8}{9} + 3\frac{3}{4} - 1\frac{5}{6} + 2\frac{5}{6}$.

37. $\frac{2}{5} - \frac{1}{35} + \frac{5}{7} - \frac{3}{15}$.

44. $5\frac{1}{8} - 2\frac{3}{8} - 1\frac{7}{8} + 4\frac{6}{16}$.

38. $\frac{5}{21} - \frac{3}{42} + \frac{6}{7} - \frac{1}{84}$.

45. $2\frac{3}{5} + 6\frac{5}{11} - 3\frac{8}{10} - 1\frac{4}{15}$.

39. $\frac{1}{6} - \frac{2}{3} + \frac{3}{4} - \frac{7}{12}$.

46. $6\frac{5}{7} + 2\frac{3}{14} - 1\frac{5}{8} + 2\frac{4}{7}$.

MULTIPLICATION OF FRACTIONS.

140. To multiply a fraction by an integer.

- At \$ $\frac{3}{4}$ a yard, what will 2 yards of cloth cost?
- If a family uses $\frac{3}{8}$ of a pound of butter each day, how much will it use in 2 days?
- If a man earns \$ $\frac{2}{10}$ an hour, how much will he earn in 3 hours?
- How much is 3 times $\frac{2}{7}$? 4 times $\frac{3}{8}$? 5 times $\frac{1}{11}$? 3 times $\frac{3}{8}$? 2 times $\frac{4}{18}$? 7 times $\frac{2}{18}$? 6 times $\frac{3}{20}$?
- How, then, may a fraction be multiplied by any integer?
- If I gave $\frac{1}{2}$ of an orange to each of 5 boys, how many halves did I give away? How many whole oranges was that?
- At \$ $\frac{3}{4}$ a yard, what will 2 yards of velvet ribbon cost?
- Mary's mother has 4 tumblers of jelly, each holding $\frac{3}{8}$ of a pint. How many pints of jelly has she?
- 5 times $\frac{3}{8}$ = ? 2 times $\frac{5}{8}$ = ? 3 times $\frac{3}{4}$ = ? 4 times $\frac{3}{8}$ = ?
2 times $\frac{4}{5}$ = ? 3 times $\frac{5}{8}$ = ? 4 times $\frac{4}{7}$ = ? 6 times $\frac{3}{10}$ = ?
- How many are 4 times $1\frac{2}{3}$? 3 times $2\frac{2}{3}$? 5 times $3\frac{1}{2}$?
- Multiply $4\frac{3}{4}$ by 5; $5\frac{1}{4}$ by 3; $9\frac{1}{8}$ by 8; $6\frac{7}{8}$ by 9.
- How much is 5 times $\frac{4}{15}$? Express the result in its smallest terms. How may this result be obtained from $\frac{4}{15}$?

13. How much is 3 times $\frac{7}{12}$? Express the result in its smallest terms. How may this result be obtained from $\frac{7}{12}$?

14. In what other way, therefore, may we sometimes multiply a fraction by an integer? Give both ways.

15. If a bookseller sells 15 books at $\$ \frac{7}{80}$ apiece, how much will he get for them?

16. When flour sells at $\$ 4\frac{1}{2}$ per barrel, how much will 9 barrels cost?

17. A clerk's wages were $\$ 10\frac{1}{4}$ per week. How much would he earn at that rate in 8 weeks?

18. If the wages of a laborer are $\$ 1\frac{1}{4}$ per day, how much does he earn in 10 days?

19. A good walker can walk $4\frac{1}{8}$ miles per hour. How far can he walk at the same rate in 8 hours?

141. It has been discovered, therefore, that:

A fraction can be multiplied by an integer by multiplying the numerator or dividing the denominator by the integer.

WRITTEN EXERCISES.

142. 1. Multiply $\frac{11}{24}$ by 8.

$$\frac{11}{24} \times 8 = \frac{11 \times 8}{24} = \frac{88}{24} = 3\frac{2}{3}, \text{ or } 3\frac{1}{3}.$$

$$\text{Or, } \frac{11}{24} \times 8 = \frac{11}{24 \div 8} = \frac{11}{3} = 3\frac{2}{3}.$$

EXPLANATION. — 8 times $\frac{11}{24}$ is $\frac{88}{24}$, or $3\frac{2}{3}$, or $3\frac{1}{3}$. Or, since dividing the denominator multiplies the fraction, 8 times $\frac{11}{24}$ is $\frac{11}{3}$, or $3\frac{2}{3}$.

Multiply:

- | | | |
|--------------------------|---------------------------|-----------------------------|
| 2. $\frac{2}{25}$ by 5. | 7. $\frac{5}{8}$ by 16. | 12. $3\frac{3}{4}$ by 6. |
| 3. $\frac{5}{22}$ by 4. | 8. $\frac{7}{10}$ by 15. | 13. $6\frac{5}{8}$ by 12. |
| 4. $\frac{11}{17}$ by 6. | 9. $\frac{11}{12}$ by 14. | 14. $10\frac{7}{12}$ by 15. |
| 5. $\frac{14}{18}$ by 5. | 10. $\frac{8}{15}$ by 20. | 15. $14\frac{3}{4}$ by 18. |
| 6. $\frac{17}{16}$ by 9. | 11. $\frac{9}{16}$ by 23. | 16. $16\frac{5}{18}$ by 8. |

17. What will 5 pounds of cheese cost at $14\frac{3}{4}$ ¢ per pound?
18. If a train goes at the rate of $20\frac{1}{2}$ miles an hour, how far will it go in 12 hours?
19. When potatoes are worth $66\frac{3}{4}$ ¢ a bushel, what must I pay for 20 bushels?
20. How much will 15 tons of coal cost at $\$6\frac{7}{10}$ a ton?
21. How many feet are there in 10 rods, since there are $16\frac{1}{2}$ feet in 1 rod?
22. A boy can chop $\frac{1}{3}$ of a cord of wood a day. How much can he chop in 3 weeks, or 18 days?
23. If a man works $8\frac{3}{4}$ hours per day, how many hours will he work in 20 days?
24. If the railroad fare between two places is $\$6\frac{1}{10}$, what will the fare of 10 persons cost?

143. To multiply an integer by a fraction.

1. James had 12 chickens, but sold $\frac{1}{3}$ of them. How many did he sell? What is $\frac{2}{3}$ of 12 chickens?
2. What will $\frac{1}{4}$ of a pound of raisins cost at 16¢ per pound? What is $\frac{3}{4}$ of 16¢?
3. Mr. Bird's horse can trot 5 miles in 20 minutes. How many minutes will it take him to go 1 mile? What is $\frac{2}{5}$ of 20? $\frac{3}{5}$ of 20? $\frac{4}{5}$ of 20?
4. What is $\frac{3}{8}$ of 24? $\frac{2}{3}$ of 14? $\frac{5}{6}$ of 18? $\frac{3}{4}$ of 28?
5. Two yards of ribbon were divided equally among 3 girls. How much did each girl get?
6. If 1 yard of braid is worth 3¢, what is $\frac{1}{4}$ of a yard worth? What is $\frac{3}{4}$ of 4¢?
7. Four quarts of milk filled 5 pitchers of equal size. How much did each pitcher hold? What is $\frac{2}{5}$ of 4? $\frac{3}{5}$ of 4? $\frac{4}{5}$ of 4?
8. What is $\frac{1}{3}$ of 3? $\frac{1}{6}$ of 2? $\frac{1}{10}$ of 7? $\frac{1}{5}$ of 3? $\frac{1}{3}$ of 5?
9. What is $\frac{1}{3}$ of 4? $\frac{2}{3}$ of 4? $\frac{3}{4}$ of 7? $\frac{1}{4}$ of 5? $\frac{3}{4}$ of 5? $\frac{1}{4}$ of 9? $\frac{1}{6}$ of 8? $\frac{5}{6}$ of 8? $\frac{5}{8}$ of 15? $\frac{1}{3}$ of 12? $\frac{2}{3}$ of 12?

10. A coat cost \$12, and a pair of trousers and a vest $\frac{2}{3}$ as much. How much did the suit cost?

11. A schoolboy paid $\frac{1}{10}$ of his money for a pencil, $\frac{2}{5}$ of it for a grammar, and $\frac{1}{5}$ of it for a penholder. If he had 40¢ before he made his purchases, how much had he left?

12. A lesson in arithmetic contained 30 examples. Henry solved $\frac{5}{6}$ of them, and his sister solved $\frac{2}{10}$ of them. How many did each solve?

13. The month of February has usually 28 days. If, in 1891, $\frac{5}{7}$ of them were stormy, how many were pleasant?

14. A paymaster paid his men only $\frac{5}{7}$ of their monthly wages. If each man earned \$35, how much did he pay to each?

144. From these examples it is seen that:

Multiplying a number by a fraction is finding a fractional part of the number.

Thus, to multiply 12 by $\frac{3}{4}$ we simply find $\frac{3}{4}$ of 12.

WRITTEN EXERCISES.

145. 1. Multiply 13 by $\frac{4}{5}$, or find $\frac{4}{5}$ of 13.

$13 \times \frac{4}{5} = \frac{13 \times 4}{5} = \frac{52}{5}$, or $10\frac{2}{5}$. EXPLANATION. — To multiply 13 by $\frac{4}{5}$ is to find $\frac{4}{5}$ of 13. $\frac{1}{5}$ of 13 is $\frac{13}{5}$, and $\frac{4}{5}$ of 13 is 4 times $\frac{13}{5}$, which is $\frac{52}{5}$, or $10\frac{2}{5}$.

When the multiplier is a mixed number, multiply by the integer and fraction separately, and add the results.

What is:	Multiply:	Multiply:
2. $\frac{3}{7}$ of 15?	8. 54 by $\frac{7}{8}$.	14. 18 by $1\frac{1}{4}$.
3. $\frac{5}{6}$ of 18?	9. 35 by $\frac{5}{6}$.	15. 25 by $3\frac{3}{4}$.
4. $\frac{3}{11}$ of 34?	10. 48 by $1\frac{1}{10}$.	16. 42 by $5\frac{3}{8}$.
5. $\frac{7}{8}$ of 24?	11. 64 by $1\frac{3}{8}$.	17. 34 by $7\frac{1}{2}$.
6. $\frac{7}{18}$ of 28?	12. 81 by $1\frac{7}{8}$.	18. 29 by $\frac{3}{4}$.
7. $\frac{2}{10}$ of 45?	13. 60 by $\frac{2}{15}$.	19. 56 by $9\frac{3}{4}$.

20. When corn is worth 75¢ a bushel, what is the value of $\frac{3}{4}$ of a bushel?

21. There are 125 acres in Mr. Swift's farm. How many acres are there in $\frac{4}{5}$ of the farm?

22. The expenses of a foundry for one month were \$583. What will $\frac{3}{4}$ of the expenses be?

23. What will $6\frac{3}{8}$ yards of silk cost at 88¢ a yard?

24. The average rate per hour of a very fast express train is 52 miles. How far will it go in $8\frac{7}{10}$ hours?

25. What will be the cost of $8\frac{1}{2}$ pounds of tea at 30¢ a pound, $6\frac{1}{2}$ pounds of sugar at 5¢ a pound, and $13\frac{1}{2}$ pounds of raisins at 16¢ a pound?

26. What will be the cost of $6\frac{3}{4}$ dozen arithmetics at \$8 per dozen, $5\frac{7}{8}$ dozen grammars at \$6 per dozen, and $6\frac{5}{8}$ dozen geographies at \$9 per dozen?

146. To multiply a fraction by a fraction.

1. If $\frac{1}{2}$ of an apple is divided into 2 equal parts, what part of the apple will each part be? How much is $\frac{1}{2}$ of $\frac{1}{2}$ of an apple?

2. If $\frac{1}{4}$ of a yard is divided into 2 equal parts, what part of a yard will each part be? How much is $\frac{1}{2}$ of $\frac{1}{4}$?

3. If $\frac{1}{8}$ of a pie is divided into 2 equal parts, what part of the pie will each part be? How much is $\frac{1}{2}$ of $\frac{1}{8}$?

4. If $\frac{1}{2}$ of a pie is divided into 3 equal parts, what part of the pie will each part be? How much is $\frac{1}{3}$ of $\frac{1}{2}$?

5. How much is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{8}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{1}{8}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{3}{8}$ of $\frac{1}{8}$? $\frac{1}{4}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{6}$ of $\frac{1}{2}$?

6. How much is $\frac{1}{5}$ of $\frac{1}{2}$? Since $\frac{1}{5}$ of $\frac{1}{2}$ is $\frac{1}{10}$, how much is $\frac{2}{5}$ of $\frac{1}{2}$? $\frac{3}{5}$ of $\frac{1}{2}$? $\frac{4}{5}$ of $\frac{1}{2}$?

7. A man having $\frac{1}{2}$ of an acre of land sold $\frac{4}{5}$ of it. What part of an acre did he sell?

8. If a yard of crape costs \$ $\frac{1}{2}$, what part of a dollar will $\frac{2}{3}$ of a yard cost?

9. I bought $\frac{1}{2}$ of a bushel of beans and planted $\frac{3}{4}$ of them. What part of a bushel did I plant?

10. How much is $\frac{1}{2}$ of $\frac{1}{3}$? Since $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$, how much is $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{2}$ of $\frac{4}{3}$? $\frac{1}{2}$ of $\frac{5}{3}$? $\frac{2}{3}$ of $\frac{2}{3}$? $\frac{2}{3}$ of $\frac{5}{3}$?

11. When cloth is worth \$ $\frac{7}{8}$ a yard, what will $\frac{1}{2}$ of a yard cost? What will $\frac{3}{4}$ of a yard cost?

12. Mr. Stone, having a lot containing $\frac{3}{4}$ of an acre, sold $\frac{1}{2}$ of it. What part of an acre did he sell? If he had sold $\frac{2}{3}$ of it, what part of an acre would he have sold?

13. If a boy who had \$ $\frac{4}{5}$ spent $\frac{1}{2}$ of it for candy, what part of a dollar did he spend? If he had spent $\frac{3}{4}$ of it, what part of a dollar would he have spent?

14. Mr. Stearns, who owned $\frac{3}{4}$ of a store, sold $\frac{3}{4}$ of his share. What part of the whole store did he sell?

15. What is $\frac{3}{4}$ of $\frac{3}{4}$? $\frac{3}{4}$ of $\frac{4}{5}$? $\frac{4}{5}$ of $\frac{2}{3}$? $\frac{3}{4}$ of $\frac{5}{6}$?

WRITTEN EXERCISES.

147. 1. Multiply $\frac{5}{8}$ by $\frac{4}{7}$, or find $\frac{4}{7}$ of $\frac{5}{8}$.

EXPLANATION.—To multiply $\frac{5}{8}$ by $\frac{4}{7}$, is to find $\frac{4}{7}$ of $\frac{5}{8}$. $\frac{1}{7}$ of $\frac{1}{8}$ is $\frac{1}{56}$ ($\frac{1}{7 \times 8}$); then, $\frac{1}{7}$ of $\frac{5}{8}$ will be ($\frac{5}{7 \times 8}$) $\frac{5}{56}$; and $\frac{4}{7}$ of $\frac{5}{8}$ will be 4 times $\frac{5}{56}$, or $\frac{20}{56}$, or $\frac{5}{14}$.

$$\frac{5}{8} \times \frac{4}{7} = \frac{5 \times 4}{8 \times 7} = \frac{5}{14}$$

148. From the above it is seen that:

In multiplying a fraction by a fraction, the numerators are multiplied together for the numerator of the product and the denominators, for its denominator.

1. Reduce any mixed numbers to improper fractions.

2. Integers may be expressed as fractions by writing 1 as a denominator. Thus, 4 may be written $\frac{4}{1}$.

3. When possible use cancellation.

4. The word *of* between the fractions is equivalent to the sign of multiplication. Such expressions are sometimes called *Compound Fractions*. Thus, $\frac{3}{4}$ of $\frac{5}{6}$ is equal to $\frac{3}{4} \times \frac{5}{6}$.

Find:

- | | | |
|-------------------------------------|-------------------------------------|---------------------------------------|
| 2. $\frac{2}{3}$ of $\frac{4}{5}$. | 6. $\frac{1}{2}$ of $\frac{5}{8}$. | 10. $\frac{2}{11}$ of $\frac{4}{5}$. |
| 3. $\frac{5}{7}$ of $\frac{2}{3}$. | 7. $\frac{1}{4}$ of $\frac{3}{8}$. | 11. $\frac{1}{2}$ of $\frac{7}{12}$. |
| 4. $\frac{2}{3}$ of $\frac{1}{4}$. | 8. $\frac{1}{2}$ of $\frac{5}{8}$. | 12. $\frac{1}{4}$ of $\frac{1}{2}$. |
| 5. $\frac{2}{3}$ of $\frac{3}{4}$. | 9. $\frac{1}{2}$ of $\frac{3}{4}$. | 13. $\frac{1}{2}$ of $\frac{1}{3}$. |

Find:

Multiply:

- | | |
|--|---|
| 14. $\frac{2}{3}$ of $4\frac{1}{2}$. | 18. $\frac{2}{3}$ of $\frac{1}{2}$ by $\frac{5}{8}$. |
| 15. $\frac{5}{8}$ of $10\frac{3}{4}$. | 19. $\frac{5}{8}$ of $\frac{7}{11}$ by $3\frac{1}{2}$. |
| 16. $\frac{7}{10}$ of $8\frac{5}{8}$. | 20. $5\frac{3}{8}$ by $\frac{2}{3}$ of $3\frac{1}{2}$. |
| 17. $\frac{1}{2}$ of $7\frac{3}{8}$. | 21. $6\frac{3}{8}$ by $\frac{4}{5}$ of $4\frac{1}{2}$. |

Find the value of:

- | | |
|--|--|
| 22. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{8}$. | 24. $\frac{5}{8} \times \frac{7}{8} \times \frac{2}{11} \times \frac{1}{4}$. |
| 23. $\frac{2}{3} \times \frac{4}{5} \times \frac{1}{8} \times \frac{1}{2}$. | 25. $2\frac{1}{2} \times \frac{3}{8} \times 1\frac{1}{2} \times \frac{3}{4}$. |

26. There are $16\frac{1}{2}$ feet in a rod. How many feet are there in $\frac{7}{8}$ of a rod? In $3\frac{1}{2}$ rods?

27. When hay is worth $\$20\frac{3}{4}$ per ton, how much will $\frac{7}{12}$ of a ton cost?

28. At $\$6\frac{2}{3}$ a ton, what must I pay for $8\frac{3}{4}$ tons of coal?

29. A man purchased $\frac{2}{3}$ of $420\frac{3}{8}$ acres of land and then sold $\frac{3}{8}$ of what he had bought. How many acres did he sell?

30. At an auction sale a cow was sold for $\$39\frac{3}{4}$, and a horse for $3\frac{3}{4}$ times as much as the cow. For how much was the horse sold?

31. If a square foot of land is worth $16\frac{3}{8}$, what is the value of $10\frac{7}{8}$ square feet?

32. The cloth of Ruth's dress cost $\$8\frac{1}{4}$. The making cost $\frac{1}{2}$ as much, the trimming cost $\frac{7}{10}$ as much as the making, and the linings cost $\frac{3}{4}$ as much as the trimmings. What did the linings cost? What was the whole cost of the dress?

DIVISION OF FRACTIONS.

149. To divide a fraction by an integer.

1. If $\frac{1}{2}$ of an orange is divided equally between 2 girls, what part of the orange will each girl have? How much is $\frac{1}{2} \div 2$?

2. A man divided $\frac{3}{4}$ of an acre into 3 equal lots. How large was each? How much is $\frac{3}{4} \div 3$?

3. If a horse ate $\frac{5}{8}$ of a bushel of oats in 4 days, how much did he eat each day? How much is $\frac{5}{8} \div 4$?

4. In dividing a fraction by an integer, what part of the fraction is divided?

5. If $\$ \frac{1}{2}$ is divided equally between 2 boys, what part of a dollar will each boy get? How much is $\frac{1}{2} \div 2$?

6. When 3 pounds of sugar can be bought for $\$ \frac{1}{2}$, what is the price per pound? How much is $\frac{1}{2} \div 3$?

7. If $\frac{1}{2}$ of a ship is owned equally by 5 men, what part of the ship is owned by each? How much is $\frac{1}{2} \div 5$?

8. In what other way, besides dividing the numerator, can a fraction be divided by an integer? State both ways in which a fraction may be divided by an integer.

9. If 3 dozen oranges can be bought for $\$ \frac{1}{2}$, what do they cost per dozen?

10. If $\frac{2}{3}$ of a pound of tea is divided equally among 3 persons, what part of a pound will each receive?

11. If $\frac{3}{4}$ of a bushel of walnuts fills 4 bags of equal size, what part of a bushel does each bag hold?

12. Mrs. Jay filled 5 tumblers with $\frac{1}{2}$ of a gallon of jelly. How much did each tumbler hold?

13. Divide $\frac{2}{7}$ by 2; $\frac{6}{7}$ by 2; $\frac{9}{10}$ by 3; $\frac{8}{11}$ by 4; $\frac{11}{18}$ by 5.

14. Divide $\frac{3}{8}$ by 2; $\frac{7}{8}$ by 3; $\frac{5}{8}$ by 4; $\frac{4}{7}$ by 5; $\frac{7}{8}$ by 6.

15. If 3 yards of cloth cost $\$ 1\frac{4}{5}$, what does 1 yard cost?

16. When 5 bushels of potatoes cost $\$ 4\frac{1}{2}$, what does a bushel cost?

17. If 3 dozen oranges cost \$ $\frac{3}{5}$, what does 1 dozen cost?
18. If 4 brooms are worth \$ $\frac{8}{10}$, how much are they worth apiece?
19. When I can buy 4 dozen eggs for \$ $\frac{8}{10}$, what is the price per dozen?
20. If 5 yards of gingham sell for \$ $\frac{4}{5}$, what is the price per yard?
21. If 6 pounds of tea are worth \$ $1\frac{1}{2}$, what is a pound worth?
22. When 5 bushels of wheat sell for \$ $4\frac{1}{2}$, what is the price per bushel?
23. James earned \$ $6\frac{1}{2}$ in 6 days. What were his daily wages?
24. A boy divided $6\frac{1}{2}$ dozen marbles equally among his 5 sisters. How many did each receive?

150. It is apparent from the solution of the above examples that:

A fraction may be divided by an integer by dividing the numerator or multiplying the denominator of the fraction by the integer.

WRITTEN EXERCISES.

- 151.** 1. Divide $\frac{20}{21}$ by 5, or find $\frac{1}{5}$ of $\frac{20}{21}$.

$$\frac{20}{21} \div 5 = \frac{20 \div 5}{21} = \frac{4}{21}.$$

EXPLANATION.—Since dividing the numerator of a fraction divides the fraction, the fraction $\frac{20}{21}$ may be divided by 5 by dividing the numerator by 5. Hence the result is $\frac{4}{21}$.

2. Divide $\frac{5}{12}$ by 6, or find $\frac{1}{6}$ of $\frac{5}{12}$.

$$\frac{5}{12} \div 6 = \frac{5}{12 \times 6} = \frac{5}{72}.$$

EXPLANATION.—Since multiplying the denominator of a fraction divides the fraction, the fraction $\frac{5}{12}$ may be divided by 6 by multiplying the denominator by 6. Hence the result is $\frac{5}{72}$.

Divide:

3. $\frac{1}{2}$ by 4.

7. $\frac{3}{4}$ by 12.

11. $\frac{1}{6}$ by 23.

4. $\frac{2}{9}$ by 5.

8. $\frac{1}{8}$ by 16.

12. $\frac{3}{5}$ by 28.

5. $\frac{3}{5}$ by 8.

9. $\frac{3}{4}$ by 18.

13. $\frac{4}{5}$ by 35.

6. $\frac{1}{4}$ by 7.

10. $\frac{3}{8}$ by 15.

14. $\frac{1}{7}$ by 42.

15. Divide $11\frac{3}{4}$ by 4.

$$11\frac{3}{4} = \frac{47}{4}.$$

$$\frac{47}{4} \div 4 = \frac{47}{16} = 2\frac{15}{16}.$$

Or,

$$\begin{array}{r} 4 \overline{)11\frac{3}{4}} \\ \underline{21\frac{5}{8}} \end{array}$$

EXPLANATION.—The mixed number may be reduced to an improper fraction, and the division performed as before.

Or, the number may be divided without being reduced to an improper fraction. Thus, 4 is contained in $11\frac{3}{4}$ 2 times, and a remainder of $3\frac{3}{4}$, or $\frac{15}{4}$; and $\frac{15}{4}$ divided by 4 equals $\frac{15}{16}$. Hence the result is $2\frac{15}{16}$.

Divide:

16. $17\frac{3}{8}$ by 7.

19. $25\frac{1}{2}$ by 8.

22. $125\frac{1}{2}$ by 7.

17. $15\frac{1}{2}$ by 4.

20. $31\frac{5}{8}$ by 6.

23. $153\frac{5}{8}$ by 8.

18. $18\frac{5}{8}$ by 5.

21. $46\frac{1}{2}$ by 9.

24. $200\frac{1}{2}$ by 9.

25. The cost of 5 tickets from Troy, N.Y., to Bethlehem, N.H., was \$40 $\frac{1}{2}$. How much did each cost?

26. The rain interfered with a laborer's working $3\frac{1}{2}$ working days in 2 weeks. How many days did he work on an average per week?

27. A farmer forgot the price which he received for his butter per pound, but he knew that he received \$2 $\frac{3}{4}$ for 8 pounds. At that rate, what did he receive per pound?

28. An estate of \$3956 $\frac{3}{8}$ was divided equally among 5 heirs. What sum did each receive?

29. The earnings of 5 men for 10 days were \$87 $\frac{1}{2}$. What were the average earnings of each per day?

30. If a man can reap a field in 10 $\frac{1}{2}$ days, in what time can 12 men reap it?

31. If a locomotive runs $142\frac{3}{4}$ miles in 6 hours, what is the average rate of speed per hour?

32. A field containing $125\frac{1}{2}$ square rods was divided into 10 lots of equal size. How large were the lots?

152. To divide an integer by a fraction.

1. How many tenths of a dollar are there in a dollar?

2. At $\$ \frac{1}{10}$ each, how many quires of paper can be bought for \$1? For \$2? For \$3? For \$4?

3. When butter is worth $\$ \frac{1}{4}$ per pound, how many pounds can be bought for \$1? For \$2? For \$3?

4. How many penknives, at $\$ \frac{1}{2}$ apiece, can be bought for \$1? For \$2? How many at $\$ \frac{2}{3}$ apiece can be bought for \$2? For \$4?

5. If a boy spends $\frac{3}{4}$ of an hour in going to school, in how many days will he spend 3 hours? 6 hours? 9 hours?

6. A man gave his daughter $\$ \frac{3}{4}$ a month to spend as she pleased. In how many months would he give her \$9?

7. The retail price of one kind of molasses is $\$ \frac{3}{4}$ per gallon. How many gallons can be bought for \$8?

8. A young man spends $\frac{3}{4}$ of each day in work and recreation. In what time will he spend 12 days in this way?

9. How many times is $\frac{1}{4}$ contained in 5? In 9? In 12?

WRITTEN EXERCISES.

153. 1. Divide 5 by $\frac{3}{7}$, or find how many times $\frac{3}{7}$ is contained in 5.

$$5 \div \frac{3}{7} = \frac{5 \times 7}{3} = 11\frac{2}{3}.$$

Or,

$$5 = \frac{35}{7}; \quad \frac{35}{7} \div \frac{3}{7} = 11\frac{2}{3}.$$

EXPLANATION. — Since $\frac{1}{7}$ is contained in 1 7 times, it is contained in 5, 5 times 7 times, or 35 times, and $\frac{3}{7}$ is contained in $5 \frac{1}{3}$ of 35 times, or $11\frac{2}{3}$ times. Or, since in 5 there are $\frac{35}{7}$, $\frac{3}{7}$ is contained in $\frac{35}{7} 11\frac{2}{3}$ times.

1. Cancellation can often be used advantageously.

2. Mixed numbers may be reduced to improper fractions.

Find the quotients of:

- | | | | |
|----------------------------|------------------------------|------------------------------|------------------------------|
| 2. $18 \div \frac{3}{4}$. | 7. $47 \div \frac{3}{4}$. | 12. $68 \div \frac{1}{8}$. | 17. $225 \div \frac{3}{8}$. |
| 3. $21 \div \frac{4}{5}$. | 8. $38 \div \frac{9}{10}$. | 13. $71 \div \frac{1}{8}$. | 18. $387 \div \frac{5}{8}$. |
| 4. $16 \div \frac{3}{8}$. | 9. $21 \div \frac{5}{7}$. | 14. $43 \div \frac{9}{10}$. | 19. $477 \div \frac{5}{7}$. |
| 5. $24 \div \frac{5}{9}$. | 10. $36 \div \frac{5}{11}$. | 15. $29 \div \frac{1}{8}$. | 20. $642 \div \frac{1}{2}$. |
| 6. $36 \div \frac{3}{8}$. | 11. $43 \div \frac{9}{8}$. | 16. $35 \div \frac{1}{7}$. | 21. $936 \div \frac{1}{7}$. |

Divide the following:

- | | | |
|----------------------------|----------------------------|----------------------------|
| 22. 29 by $4\frac{1}{4}$. | 24. 36 by $8\frac{1}{4}$. | 26. 33 by $7\frac{3}{4}$. |
| 23. 32 by $3\frac{1}{2}$. | 25. 91 by $5\frac{1}{2}$. | 27. 54 by $6\frac{1}{8}$. |

28. Divide 24 by $5\frac{1}{4}$.

54) 24

7 7

39) 168 ($4\frac{4}{9}$

EXPLANATION. — By changing the dividend and divisor to 7ths, the division is performed as in simple division.

Divide the following:

- | | | |
|-----------------------------|-----------------------------|-----------------------------|
| 29. 36 by $7\frac{3}{4}$. | 32. 59 by $4\frac{3}{4}$. | 35. 39 by $7\frac{3}{4}$. |
| 30. 46 by $3\frac{5}{8}$. | 33. 36 by $3\frac{5}{8}$. | 36. 35 by $16\frac{1}{2}$. |
| 31. 52 by $6\frac{7}{12}$. | 34. 96 by $4\frac{7}{12}$. | 37. 46 by $8\frac{3}{4}$. |

38. When cloth is \$.10 $\frac{3}{4}$ per yard, how many yards can be bought for \$3.36?

39. My lot has a frontage on the street of 385 feet. How many rods is it in front, 16 $\frac{1}{2}$ feet being a rod?

40. A cheap kind of flour sells for \$3 $\frac{5}{8}$ per barrel. How many barrels can be bought for \$76 $\frac{1}{2}$?

41. A farmer bought a wagon for \$68, paying for it in apples at \$1 $\frac{7}{8}$ per barrel. How many barrels did he give for it?

42. Boots are commonly sold in cases containing 12 pairs. If a dealer paid \$99 for a quantity of boots at \$2 $\frac{1}{4}$ a pair, how many cases did he purchase?

154. To divide a fraction by a fraction.

1. At $\$ \frac{1}{2}$ each, how many books can be bought for $\$ \frac{4}{3}$?
At $\$ \frac{2}{3}$?

2. Among how many children can $\frac{6}{8}$ of a melon be divided, if each child is given $\frac{1}{8}$? $\frac{2}{8}$? $\frac{3}{8}$?

3. If cream is worth $\$ \frac{1}{10}$ per pint, how many pints can be bought for $\$ \frac{8}{10}$? If $\$ \frac{2}{10}$ per pint? If $\$ \frac{4}{10}$?

4. How many pitchers, each holding $\frac{2}{3}$ of a gallon, can be filled from $\frac{10}{3}$ of a gallon?

5. If a man uses $\frac{3}{16}$ of a pound of butter each day, how many days will $\frac{12}{16}$ of a pound last him?

6. How many times is $\frac{2}{3}$ contained in $\frac{10}{3}$? $\frac{4}{3}$ in $\frac{8}{3}$? $\frac{3}{3}$ in $\frac{9}{3}$? $\frac{3}{3}$ in $\frac{12}{3}$ (or 1)? $\frac{6}{3}$ in 1? $\frac{2}{3}$ in 1? $\frac{4}{3}$ in 1? $\frac{1}{3}$ in $\frac{9}{3}$? $\frac{2}{3}$ in $\frac{6}{3}$? $\frac{3}{3}$ in $\frac{6}{3}$?

7. If a train moves $\frac{2}{3}$ of a mile each minute, how many minutes will it require to go $3\frac{1}{3}$ miles? How many thirds are there in $3\frac{1}{3}$ miles?

8. If it takes $\frac{3}{8}$ of a yard of lace to trim a bonnet, how many bonnets can be trimmed with $2\frac{2}{8}$ yards of lace?

9. How many aprons can be made from $4\frac{1}{2}$ yards of muslin, if each apron requires $1\frac{1}{2}$ yards?

How many half yards are there in $1\frac{1}{2}$ yards?

How many half yards are there in $3\frac{1}{2}$ yards?

10. How many times are 3 half yards contained in 6 half yards?

11. How many books can be bought with $\$ 6\frac{3}{4}$, at $\$ 1\frac{1}{4}$ apiece?

12. How many pitchers, each holding $2\frac{3}{4}$ pints, will be required to hold $13\frac{3}{4}$ pints of milk?

13. Among how many children can $\frac{1}{2}$ of a pound of candy be divided, if each child is given $\frac{1}{8}$ of a pound?

14. How many eighths are there in $\frac{1}{2}$?

15. If a coat requires $\frac{3}{4}$ of a dozen buttons, how many coats will require $4\frac{1}{2}$ dozen buttons?

16. With \$ $\frac{3}{4}$, how many yards of cloth can be bought at \$ $\frac{3}{8}$ per yard?

WRITTEN EXERCISES.

155. 1. Divide $\frac{7}{8}$ by $\frac{2}{3}$, or find how many times $\frac{2}{3}$ is contained in $\frac{7}{8}$.

$$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16}.$$

EXPLANATION. — Since it is required to find how many times $\frac{2}{3}$ is contained in $\frac{7}{8}$, we first find how many times $\frac{2}{3}$ is contained in 1, and then find $\frac{7}{8}$ of that result.

Since $\frac{1}{3}$ is contained in 1 3 times, $\frac{2}{3}$ is contained in 1 one half of 3 times, or $\frac{3}{2}$ times.

Since $\frac{2}{3}$ is contained in 1 $\frac{3}{2}$ times, in $\frac{7}{8}$ it will be contained $\frac{7}{8}$ of $\frac{3}{2}$ times, or $\frac{21}{16}$ times.

156. From the explanation given, it is apparent that:

A fraction may be divided by a fraction by multiplying the dividend by the divisor inverted.

1. Use cancellation whenever it is possible to do so.
2. Reduce mixed numbers and integers to improper fractions.

2. Divide $\frac{3}{4}$ by $\frac{5}{7}$.

SOLUTION. — $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$. Or, $\frac{3}{4} = \frac{3}{2} \times \frac{1}{2}$, and $\frac{5}{7} = \frac{5}{2} \times \frac{1}{2}$. Therefore, $\frac{3}{2} \div \frac{5}{2} = \frac{3}{5}$.

157. Hence it is seen that:

Another method of dividing a fraction by a fraction is to reduce the fractions to similar fractions, and then to divide the numerator of the dividend by the numerator of the divisor.

Divide:

- | | | |
|---|--|--|
| 3. $\frac{7}{8}$ by $\frac{6}{11}$. | 9. $8\frac{1}{2}$ by $\frac{4}{5}$. | 15. $26\frac{1}{4}$ by $4\frac{3}{4}$. |
| 4. $1\frac{4}{5}$ by $\frac{7}{8}$. | 10. $3\frac{7}{12}$ by $1\frac{1}{2}$. | 16. $19\frac{3}{4}$ by $3\frac{3}{4}$. |
| 5. $\frac{8}{9}$ by $\frac{3}{4}$. | 11. $9\frac{9}{14}$ by $2\frac{3}{4}$. | 17. $36\frac{1}{4}$ by $1\frac{9}{10}$. |
| 6. $1\frac{5}{12}$ by $1\frac{2}{15}$. | 12. $7\frac{6}{11}$ by $4\frac{1}{2}$. | 18. $18\frac{3}{4}$ by $\frac{3}{5}$. |
| 7. $\frac{9}{10}$ by $\frac{7}{8}$. | 13. $5\frac{3}{4}$ by $\frac{5}{6}$. | 19. $16\frac{3}{5}$ by $3\frac{5}{6}$. |
| 8. $1\frac{1}{6}$ by $2\frac{9}{10}$. | 14. $12\frac{2}{3}$ by $7\frac{5}{12}$. | 20. $7\frac{1}{4}$ by $1\frac{3}{8}$. |

21. How many times may $2\frac{1}{2}$ gallons be drawn from a barrel that holds $31\frac{1}{2}$ gallons?

22. How many bushels of potatoes, at $\$1\frac{1}{8}$ a bushel, may be bought with \$5?

23. How many barrels of apples, each holding $2\frac{3}{8}$ bushels, can be filled with 18 bushels?

24. At $\$1\frac{1}{8}$ a basket for peaches, how many baskets of peaches shall I get for \$8?

25. When flour is $\$7\frac{3}{8}$ per barrel, how many barrels of flour can be bought for \$59?

26. How many pounds of beef, at $11\frac{3}{4}$ ¢ per pound, can be bought for $87\frac{1}{2}$ ¢?

27. How many books, at $\$3\frac{1}{8}$ per volume, can be purchased for $\$31\frac{1}{4}$?

28. When oranges are $12\frac{1}{2}$ ¢ per dozen, how many dozen can be bought for $98\frac{3}{4}$ ¢?

29. How many plows, at $\$11\frac{1}{8}$ each, can be bought for $\$67\frac{1}{8}$?

30. How many acres of land, at $\$43\frac{1}{2}$ per acre, can be bought for $\$237\frac{1}{2}$?

31. A railroad $16\frac{1}{2}$ miles long cost \$66,937. What was the cost per mile?

32. If $4\frac{1}{2}$ hogsheads hold $25\frac{7}{8}$ bushels, what will 1 hogshead hold?

33. If a field containing $54\frac{1}{2}$ acres yielded 1165 bushels of rye, what was the average crop per acre?

34. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{7}{9}$ by $\frac{6}{5}$ of $\frac{2}{3}$ of $\frac{5}{8}$.

$$\left(2\frac{3}{4} \times \frac{5}{7} \times \frac{7}{9}\right) \div \left(6\frac{2}{5} \times \frac{2}{3} \times \frac{5}{8}\right) = \frac{2}{3} \times \frac{3}{4} \times \frac{5}{7} \times \frac{7}{9} \times \frac{5}{6} \times \frac{3}{2} \times \frac{8}{5} = \frac{5}{9}.$$

EXPLANATION. — In solving examples like this, *all the factors of the divisor should be inverted*. All the integers and mixed numbers should be changed to improper fractions before the factors of the divisor are inverted. Use cancellation to abridge the process.

Divide:

35. $\frac{3}{8}$ of $\frac{4}{7}$ of $\frac{2}{3}$ of $\frac{5}{8}$ by $\frac{5}{8}$ of $\frac{7}{8}$ of $\frac{4}{11}$ of $\frac{2}{3}$ of 6.
36. $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{2}{3}$ of $\frac{7}{15}$ by $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{2}{3}$ of 8.
37. $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{1}{15}$ of $\frac{5}{12}$ by $\frac{1}{7}$ of $\frac{2}{3}$ of $\frac{7}{12}$ of 24.
38. $\frac{7}{10}$ of $\frac{5}{7}$ of $\frac{2}{3}$ of $\frac{1}{4}$ by $\frac{3}{12}$ of $\frac{5}{7}$ of $\frac{1}{15}$ of 36.
39. $\frac{5}{8}$ of $\frac{5}{7}$ of $\frac{1}{15}$ of $\frac{2}{18}$ by $\frac{2}{11}$ of $\frac{1}{15}$ of $\frac{2}{3}$ of $\frac{4}{15}$ of 30.
40. $\frac{7}{12}$ of $\frac{5}{7}$ of $\frac{1}{15}$ of $\frac{2}{10}$ by $\frac{5}{7}$ of $\frac{1}{10}$ of $\frac{2}{11}$ of $\frac{1}{2}$ of 44.
41. $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{1}{9}$ of $\frac{2}{3}$ by $\frac{1}{7}$ of $\frac{5}{21}$ of $\frac{1}{2}$ of $\frac{2}{10}$ of 25.
42. $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{1}{2}$ of $\frac{4}{25}$ by $\frac{2}{8}$ of $\frac{4}{10}$ of $\frac{2}{7}$ of $\frac{2}{4}$ of 16.
43. $\frac{2}{7}$ of $\frac{1}{10}$ of $\frac{1}{2}$ of $\frac{2}{7}$ by $\frac{4}{7}$ of $\frac{4}{10}$ of $\frac{1}{2}$ of $\frac{2}{12}$ of 10.
44. $\frac{1}{12}$ of $\frac{2}{7}$ of $\frac{2}{3}$ of $\frac{7}{8}$ by $\frac{5}{8}$ of $\frac{2}{3}$ of $\frac{1}{7}$ of $\frac{2}{4}$ of 20.
45. $\frac{2}{8}$ of $\frac{5}{7}$ of $\frac{1}{7}$ of $\frac{7}{15}$ by $\frac{5}{9}$ of $\frac{1}{2}$ of $\frac{1}{10}$ of $\frac{5}{11}$ of 45.
46. $\frac{7}{10}$ of $\frac{2}{3}$ of $\frac{2}{7}$ of $\frac{5}{9}$ by $\frac{2}{8}$ of $\frac{1}{7}$ of $\frac{2}{15}$ of $\frac{1}{10}$ of 18.

158. Expressions of unexecuted division of fractions are frequently written in the form of fractions. Such expressions are sometimes called **Complex Fractions**.

Thus, $\frac{2}{3} \div \frac{5}{7}$ may be written $\frac{\frac{2}{3}}{\frac{5}{7}}$; $6\frac{2}{3} \div \frac{5}{8}$, $\frac{6\frac{2}{3}}{\frac{5}{8}}$.

1. Find the value of the fractional form, $\frac{\frac{4}{5}}{\frac{3}{11}}$

SOLUTION. — $\frac{\frac{4}{5}}{\frac{3}{11}} = \frac{4}{5} \div \frac{3}{11} = \frac{4}{5} \times \frac{11}{3} = \frac{44}{15} = 2\frac{14}{15}$.

Find the value of the following:

- | | | | |
|--------------------------------------|---------------------------------------|---|---|
| 2. $\frac{\frac{3}{5}}{\frac{2}{3}}$ | 5. $\frac{5\frac{1}{2}}{4}$ | 8. $\frac{6\frac{1}{2}}{2\frac{1}{4}}$ | 11. $\frac{7\frac{2}{3}}{6\frac{2}{3}}$ |
| 3. $\frac{\frac{4}{5}}{\frac{3}{4}}$ | 6. $\frac{4\frac{1}{2}}{\frac{2}{3}}$ | 9. $\frac{1\frac{2}{3}}{1\frac{1}{2}}$ | 12. $\frac{\frac{2}{3} \text{ of } 8}{5\frac{1}{2}}$ |
| 4. $\frac{\frac{6}{7}}{\frac{5}{8}}$ | 7. $\frac{5}{\frac{3}{7}}$ | 10. $\frac{5\frac{3}{8}}{3\frac{1}{8}}$ | 13. $\frac{2\frac{2}{10}}{\frac{1}{2} \text{ of } \frac{4}{5}}$ |

159. To find the relation of one number to another.

1. There were 5 oranges on a plate. Willie took 1 orange, and Eddie took 2. What part of all the oranges did Willie take? Eddie?

2. What part of 8¢ is 1¢? 3¢? 5¢?

7 is $\frac{7}{10}$ of 10, because it is 7 of the 10 equal parts into which 10 may be divided. What part of 10 is 9?

3. What part of 10 acres are 5 acres? Of 9 quarts are 3 quarts? Of 8 pounds are 2 pounds? Of 10 inches are 2 inches?

4. 1 is what part of 12? 2 of 12? 3 of 12? 8 of 12?

5. 5 melons are what part of 7 melons?

6. \$5 are what part of \$8?

7. 2 fifths are what part of 3 fifths?

8. 3 eighths are what part of 7 eighths?

9. $\frac{2}{3}$ is what part of $\frac{5}{6}$? $\frac{3}{10}$ of $\frac{9}{10}$? $\frac{5}{18}$ of $\frac{11}{18}$?

10. What part of 2 ounces is $\frac{3}{4}$ of an ounce?

SUGGESTION.—2 ounces = $\frac{8}{4}$ ounces. What part of $\frac{8}{4}$ is $\frac{3}{4}$?

11. What part of \$5 are \$2 $\frac{1}{2}$?

12. What part of $\frac{1}{2}$ of a gallon is $\frac{1}{8}$ of a gallon?

SUGGESTION.— $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{8} = \frac{1}{8}$. What part of $\frac{3}{6}$ is $\frac{1}{8}$?

13. What part of 10 quarts are 3 $\frac{1}{2}$ quarts?

14. What part of 7 acres are 1 $\frac{4}{5}$ acres?

15. What part of $\frac{7}{8}$ of a rod is $\frac{1}{4}$ of a rod?

16. What part of 2 $\frac{3}{8}$ pounds are 1 $\frac{1}{4}$ pounds?

17. What part of 10 is $\frac{1}{5}$? Of 10 is $\frac{2}{5}$? Of 10 is 1 $\frac{1}{2}$?

WRITTEN EXERCISES.

160. 1. What part of \$100 are \$72?

EXPLANATION.—\$1 is $\frac{1}{100}$ of \$100; consequently, $\frac{72}{100} = \frac{18}{25}$. \$72 are $\frac{72}{100}$ of \$100, or $\frac{18}{25}$ of \$100.

1. When one or both numbers are fractions not having a common denominator, the numbers should be reduced to equivalent fractions having a common denominator.

2. Mixed numbers should be reduced to improper fractions.

What part of:

2. 85 is 30?

7. 25 is $2\frac{1}{2}$?

12. $1\frac{1}{2}$ is $\frac{2}{3}$?

3. 175 is 28?

8. 40 is $3\frac{3}{4}$?

13. $\frac{1}{12}$ is $\frac{1}{20}$?

4. 12 is $\frac{3}{8}$?

9. 64 is $4\frac{1}{6}$?

14. $3\frac{1}{12}$ is $1\frac{1}{2}$?

5. 20 is $\frac{4}{7}$?

10. 33 is $1\frac{7}{8}$?

15. $7\frac{1}{3}$ is $4\frac{1}{3}$?

6. 32 is $\frac{5}{9}$?

11. 72 is $2\frac{3}{4}$?

16. $5\frac{7}{8}$ is $3\frac{3}{4}$?

17. In a farm of 120 acres there were $35\frac{1}{4}$ acres of pasture land. What part of the farm was used for pasture?

18. What part of $272\frac{1}{4}$ square feet are 9 square feet?

19. If a boy can do a piece of work in 25 days, what part of the work can he do in $7\frac{2}{3}$ days?

20. Miss Reed paid \$ $20\frac{3}{4}$ for a gown, and \$ $6\frac{1}{2}$ for a hat. What part of the price of the dress did the hat cost?

21. If the working day is 10 hours long, how much will a man who earns \$3 a day get for 6 hours' work?

ALIUOT PARTS.

161. An **aliuot part** of a number is any integer or mixed number that will exactly divide it.

ALIUOT AND OTHER PARTS OF A DOLLAR.

$50¢ = \$\frac{1}{2}.$

$10¢ = \$\frac{1}{10}.$

$66\frac{2}{3}¢ = \$\frac{2}{3}.$

$33\frac{1}{3}¢ = \$\frac{1}{3}.$

$8\frac{1}{3}¢ = \$\frac{1}{12}.$

$75¢ = \$\frac{3}{4}.$

$25¢ = \$\frac{1}{4}.$

$6\frac{1}{4}¢ = \$\frac{1}{16}.$

$40¢ = \$\frac{2}{5}.$

$20¢ = \$\frac{1}{5}.$

$5¢ = \$\frac{1}{20}.$

$83\frac{1}{3}¢ = \$\frac{5}{6}.$

$16\frac{2}{3}¢ = \$\frac{1}{6}.$

$4¢ = \$\frac{1}{25}.$

$37\frac{1}{2}¢ = \$\frac{3}{8}.$

$12\frac{1}{2}¢ = \$\frac{1}{8}.$

$2¢ = \$\frac{1}{50}.$

$87\frac{1}{2}¢ = \$\frac{7}{8}.$

162. 1. What will 12 pounds of butter cost at $33\frac{1}{3}¢$ per pound?

SOLUTION. — $33\frac{1}{3}¢ = \$\frac{1}{3}$. Since 1 pound costs $\$ \frac{1}{3}$, 12 pounds will cost 12 times $\$ \frac{1}{3}$, which is $\$ 1^2$, or \$4.

2. What will 20 readers cost at 25¢ apiece? At 50¢?
3. What will 25 sponges cost at 20¢ apiece?
4. What will 36 yards of ribbon cost at $16\frac{2}{3}$ ¢ a yard?
5. What will 48 yards of lace cost at $12\frac{1}{2}$ ¢ a yard?

What is the cost of:

6. 120 pounds of sugar at 5¢? At $8\frac{1}{3}$ ¢? At 10¢?
7. 12 neckties at 75¢? At $66\frac{2}{3}$ ¢? At $83\frac{1}{3}$ ¢?
8. 40 baskets at $37\frac{1}{2}$ ¢? At 40¢? At 30¢?
9. 80 gallons of molasses at $37\frac{1}{2}$ ¢? At $62\frac{1}{2}$ ¢? At $87\frac{1}{2}$ ¢?
10. 28 dozen pencils at $12\frac{1}{2}$ ¢? At $16\frac{2}{3}$ ¢? At 20¢?
11. 16 pairs of gloves at $66\frac{2}{3}$ ¢? At $83\frac{1}{3}$ ¢? At 75¢?
12. 24 days' work at \$1.25? At \$1.33 $\frac{1}{3}$? At \$1.16 $\frac{2}{3}$?
13. At $16\frac{2}{3}$ ¢ per pound, how many pounds of raisins can be bought with \$2?

SOLUTION. — $16\frac{2}{3}$ ¢ = $\$ \frac{1}{3}$. Since 1 pound costs $\$ \frac{1}{3}$, as many pounds can be bought with \$2 as $\$ \frac{1}{3}$ is contained times in \$2, or $\$ \frac{1}{3}$ ², which is 12 times. Therefore, 12 pounds of raisins can be bought with \$2.

14. At $33\frac{1}{3}$ ¢ a dozen, how many dozen eggs can I buy with \$5? \$3? \$7? \$4?

15. At $37\frac{1}{2}$ ¢ apiece, how many books can be bought for \$2 $\frac{1}{4}$? \$3 $\frac{3}{4}$? \$4 $\frac{1}{2}$? \$5 $\frac{1}{4}$?

16. For \$2 $\frac{1}{2}$, how many yards of muslin can be bought at 10¢ a yard? At $12\frac{1}{2}$ ¢? At $16\frac{2}{3}$ ¢?

163. To find the whole when a part and the relation of the part to the whole are given.

1. If \$5 is $\frac{1}{2}$ of my money, how much money have I? 10 cents is $\frac{1}{4}$ of my brother's money. How much has he?

2. A man sold 5 gallons of vinegar, which was $\frac{1}{3}$ of all he had. How many gallons had he?

3. A farmer sold 10 quarts of milk per day, which was $\frac{1}{5}$ of what the cows produced. How much did they produce per day?

4. A boy paid \$6 for a coat, and that sum was $\frac{1}{4}$ of his wages for a month. What were his monthly wages?

5. A lad purchased a box of cartridges, fired off 10 of them, and had $\frac{9}{10}$ of them left. How many cartridges were there in the box?

6. A boy borrowed 50 feet of kite string, which was $\frac{1}{10}$ of what another boy had. How many feet of string did the other boy have?

7. A farmer estimated that a hail storm had caused him a loss of 5 bushels of oats per acre, or $\frac{1}{10}$ of his crop. How much did he think his land would yield him per acre?

8. A boy and his sister played backgammon for some time, his sister beating him 15 games, which was $\frac{1}{4}$ of the number of games played. How many did they play?

9. If 2 thirds of a number is 6, how much is $\frac{1}{3}$ of it?

10. If 3 fourths of a number is 9, how much is $\frac{1}{4}$ of it?

11. If 5 sevenths of a number is 10, how much is $\frac{1}{7}$ of it? Since $\frac{1}{7}$ of the number is 2, what is the number?

12. If $\frac{2}{3}$ of a number is 8, how much is $\frac{1}{3}$ of the number? Since $\frac{1}{3}$ of the number is 4, what is the number?

WRITTEN EXERCISES.

164. 1. 320 is $\frac{5}{8}$ of what number?

$$\frac{5}{8} = 320.$$

$$\frac{1}{8} = 64.$$

$$\text{Number} = 512.$$

EXPLANATION.—Since 320 is $\frac{5}{8}$ of a certain number, $\frac{1}{8}$ of 320, or 64, is $\frac{1}{8}$ of the number; and since 64 is $\frac{1}{8}$ of the number, the number must be 8 times 64, or 512. Hence 320 is $\frac{5}{8}$ of 512.

Find the number of which

2. 40 is $\frac{5}{8}$.

8. 125 is $\frac{5}{16}$.

14. $\frac{5}{8}$ is $\frac{1}{8}$.

3. 60 is $\frac{5}{6}$.

9. 340 is $\frac{1}{20}$.

15. $\frac{7}{20}$ is $\frac{7}{5}$.

4. 85 is $\frac{5}{10}$.

10. 560 is $\frac{1}{16}$.

16. $\frac{1}{16}$ is $\frac{7}{8}$.

5. 75 is $\frac{3}{4}$.

11. 300 is $\frac{6}{17}$.

17. $\frac{2}{7}$ is $\frac{5}{8}$.

6. 65 is $\frac{5}{7}$.

12. 240 is $\frac{1}{12}$.

18. $\frac{8}{7}$ is $\frac{9}{8}$.

7. 81 is $\frac{9}{10}$.

13. 330 is $\frac{1}{11}$.

19. $\frac{4}{11}$ is $\frac{9}{10}$.

20. A boy sold his bicycle at second hand for $\frac{2}{3}$ of the cost, receiving for it \$80. How much did it cost him?

21. Our tennis court is just $\frac{2}{3}$ of what it was originally, and it is now 120 feet. How long was it originally?

22. The distance between Rochester and Syracuse, N.Y., is 81 miles, and it lacks 4 miles of being $\frac{1}{5}$ of the distance from New York to Buffalo. What is the distance from New York to Buffalo?

23. The width of a river near its source was 45 feet, and that distance was $\frac{3}{10}$ of what it was 20 miles farther down. How wide was it at the latter point?

24. A gardener set out some cabbage plants, but only 80 of them lived, which was just $\frac{5}{12}$ of the number set out. How many did he set out?

25. A way train stopping at all stations averaged 25 miles per hour; but the rate of the train was only $\frac{5}{11}$ of that of a fast express train. How many miles per hour did the express train run?

REVIEW EXERCISES.

ORAL EXERCISES.

165. 1. Change to improper fractions: $3\frac{2}{3}$; $5\frac{2}{3}$; $6\frac{2}{3}$; $5\frac{1}{3}$; $7\frac{2}{3}$.

2. Change to mixed numbers: $\frac{27}{8}$; $\frac{38}{5}$; $\frac{41}{7}$; $\frac{52}{8}$; $\frac{39}{4}$; $\frac{42}{6}$; $\frac{57}{9}$.

3. Find the sum of $3\frac{1}{2}$, $4\frac{3}{4}$, $5\frac{1}{4}$, $6\frac{1}{2}$, $7\frac{3}{4}$, and $3\frac{3}{4}$.

4. Find the sum of $6\frac{1}{4}$, $4\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{5}{8}$, $4\frac{1}{8}$, and $2\frac{3}{8}$.

5. A man had 3 lots containing respectively $3\frac{1}{2}$ acres, $2\frac{1}{4}$ acres, and $3\frac{1}{8}$ acres. How many acres did they all contain?

6. The expense of a carpet was $\$12\frac{3}{4}$, and it cost $\$3\frac{1}{2}$ to put it down. What was the entire cost of the carpet laid in the room?

7. Terrence's uncle gave him \$10 on Christmas, but he spent $3\frac{1}{2}$ of it. How much had he left?

8. A father allowed his son \$40 per year for spending money. At the end of the year the boy had $18\frac{1}{2}$. How much did he spend?

9. The expense of a Columbia bicycle was \$140; but a boy bought one that pleased him just as well for $\frac{3}{4}$ of that price. How much did it cost him?

10. The price of California peaches was $\$ \frac{3}{4}$ per dozen in July. How much did 8 dozen cost?

11. A man had a field containing $4\frac{2}{3}$ acres, which he divided into lots containing $\frac{7}{8}$ of an acre. How many lots were there?

12. A wagon which cost $\$ 50\frac{1}{2}$ was sold for $\$ 46\frac{1}{2}$. How much was the loss?

13. After purchasing a silver watch for $\$ 19\frac{1}{2}$, I had $\$ 21\frac{1}{2}$ left. How much had I at first?

14. A man owed for groceries $\$ 18\frac{1}{2}$; for meat, $\$ 10\frac{1}{2}$, and for clothes $\$ 8\frac{1}{2}$. If he paid the debts out of \$50, which he has, how much did he have left?

15. A boy went on a visit to his cousins with \$35 to pay his expenses. On returning he found that he had spent $\frac{5}{7}$ of his money. How much had he left?

16. The proprietor of a boarding house, after buying provisions for the day, found that he had $\$ 35\frac{1}{4}$ left, which was just $\frac{1}{4}$ of the money he had before making the purchases. How much did he spend?

17. An American flag was purchased by some school children for $\$ 11\frac{3}{8}$, and the flag-staff cost $\$ 6\frac{1}{4}$. What did both cost?

18. A man sold a horse for \$160 and a cow for $\frac{2}{3}$ as much. How much did he get for both?

19. A cow was sold for \$60, which was $\frac{3}{8}$ of what a horse was sold for. How much was received for both?

20. After spending $\frac{3}{4}$ of my money, I had \$35 left. How much had I?

21. A horse traveled $38\frac{1}{2}$ miles in 7 hours. What was his average rate of speed per hour?

22. A man sold a cow for \$40 and spent $\frac{1}{5}$ of the sum. Afterwards he bought a horse and paid in cash what he had left from the sale of the cow. What did the horse cost him, if the cash payment was $\frac{1}{5}$ of the cost of the horse?

23. A farm was sold in 2 parts. The first part contained 50 acres and that was $\frac{5}{7}$ of the second part. How many acres were there in the farm?

24. I spent $\frac{1}{3}$ of my money for books, $\frac{1}{3}$ of it for furniture, and had \$100 left. How much had I at first?

25. James had a sum of money such that $\frac{1}{2}$ of it was just $\frac{1}{8}$ of Henry's. What part of Henry's was the whole of James's?

26. One half of a mother's age was $\frac{5}{8}$ of her daughter's. What part of the mother's age was the daughter's?

27. What part of the daughter's age was the mother's?

28. $\frac{5}{8}$ multiplied by another fraction gives a product of $\frac{5}{8}$. What is the multiplier?

29. The difference between $\frac{1}{3}$ and $\frac{1}{4}$ of a number is 8. What is the number?

WRITTEN EXERCISES.

166. Find the value of:

- $3\frac{1}{2} + 3\frac{4}{7} + 6\frac{5}{8} - 3\frac{2}{3} - \frac{5}{7} - 5\frac{5}{18}$.
- $5\frac{1}{3} + 6\frac{4}{5} + 3\frac{7}{10} - 6\frac{2}{3} - 5\frac{4}{15} + 2\frac{2}{5}$.
- $7\frac{3}{4} + 5\frac{2}{3} - 9\frac{7}{12} + 8\frac{1}{5} - \frac{1}{5} + 5\frac{3}{10}$.
- $(7\frac{5}{8} + 3\frac{2}{5}) \times 5\frac{1}{2} + (3\frac{3}{4} + 1\frac{9}{10}) \times 3\frac{1}{4}$.
- $(3\frac{2}{3} - 1\frac{1}{4}) \div 1\frac{2}{5} + (2\frac{1}{2} - 1\frac{1}{4}) \times 6\frac{1}{2} - 5$.
- $(4\frac{2}{3} + 6\frac{1}{4} - 3\frac{2}{5}) \div \frac{1}{10} + (3\frac{1}{8} - 2\frac{1}{4}) \times 6$.
- $(4\frac{1}{4} - 3\frac{1}{8} + 7\frac{1}{10} + \frac{3}{5} + 4) \div \frac{1}{80} + 8$.
- $(6\frac{2}{3} + 4\frac{2}{3}) \times (5\frac{1}{3} - 3\frac{1}{4}) \div (2\frac{1}{5} - 1\frac{2}{3})$.

9. How much will 5 masons earn in $4\frac{1}{2}$ days, at $\$3\frac{1}{4}$ per day?

10. After selling $\frac{1}{8}$ of his load of potatoes to one person, $\frac{1}{4}$ to another, and 25 bushels to another, a farmer had sold his entire load. How many bushels did he sell?

11. One fifth of the expenses of an excursion were paid by Mr. A., $\frac{1}{8}$ of them by Mr. B., and the rest, which was \$17, by the rest of the party. What were the expenses of the excursion?

12. A man forgot how many bushels of rye he had sold, but he remembered that he had received \$185 for the rye at $\$7\frac{1}{4}$ per bushel. How many bushels did he sell?

13. A man loaned some money, receiving at the end of the year \$245 interest for it. If the rate of interest was $\frac{1}{100}$ of the sum loaned, how much did he loan?

14. A man finds that he must save \$600 per year. He takes boarders, and can accommodate 10. Suppose that he pays \$2520 for provisions, servants, and rent, and his rooms are full all the year; how much must he charge per week to save \$600?

15. A man of means spent $\frac{1}{3}$ of his life traveling in America, $\frac{1}{4}$ of it in Europe, and the rest of it at his own home. What part of his life did he spend at his own home?

16. If he spent 30 years at his own home, how old was he?

17. If A can do a piece of work in 6 days, what part of it can he do in 1 day? If B can do the same piece of work in 7 days, what part of it can he do in 1 day? If both work together, what part of it can they do in 1 day?

18. If both, working together, could only do $\frac{1}{42}$ of it in 1 day, how many days would they need to complete it? Since both together can do $\frac{1}{42}$ in 1 day, what part of 42 days will they need to complete it?

19. A man bequeathed $\frac{1}{4}$ of his property to his brother, $\frac{1}{8}$ of it to his sister, and the rest, which was \$62500, to a college. How much did he bequeath in all?

20. A can do a piece of work in 3 days, and B can do it in 5 days. In what time can both together do it?

21. A can do a piece of work in 8 days, and B can do it in 10 days. In what time can both do it together?

22. It takes A and B 8 days to do a piece of work which B can do alone in 12 days. In what time can A do it?

23. Two men during a year earned together \$1085; but one of them only earned $\frac{3}{4}$ as much as the other. How much did each earn?

24. Two brothers earn respectively \$2 and \$3 per day. In what time can both together earn \$13?

25. A man who owned $\frac{2}{3}$ of a sash factory sold $\frac{3}{8}$ of his share for \$24000. How much was the factory worth at that rate?

26. Divide $\frac{3}{8}$ of $\frac{5}{7}$ of $\frac{11}{16}$ of $4\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{8}{9}$ of $\frac{6}{11}$ of $\frac{1}{10}$.

27. Divide $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ of $6\frac{1}{4}$ by $\frac{7}{16}$ of $\frac{3}{4}$ of $\frac{1}{3}$ of $2\frac{3}{4}$.

28. Find the sum and the difference of $4\frac{1}{4}$ and $3\frac{5}{8}$; divide the sum by the difference; the difference by the sum; and then find the product of the quotients.

29. Multiply $3\frac{5}{8}$ by $2\frac{3}{8}$; divide $3\frac{5}{8}$ by $2\frac{3}{8}$; and find the difference between the two results.

30. A man spent $\frac{1}{3}$ of his money and \$25 more, and then had \$225 left. How much money had he?

31. A man after spending \$2250 in repairing his house, found that it cost him, including repairs, just $\frac{5}{4}$ of the original cost. What was the original cost?

32. The estimated cost of a building was $\frac{3}{4}$ of its actual cost. If $\frac{1}{4}$ of the estimated expense was \$5000, what was the actual cost of the building?

33. A school in one of our cities had the names of 1039 pupils on its rolls. If there had been one girl more, there

would have been $2\frac{1}{4}$ times as many girls as boys in the school. How many boys and how many girls were enrolled?

34. A man engaged in manufacturing made a profit of $\frac{5}{100}$ on his capital, and this sum, added to his capital, made \$27060. How much capital had he invested in the business?

35. The product of three numbers is $72\frac{1}{2}$. Two of them are $5\frac{3}{8}$ and $9\frac{5}{8}$. What is the other number?

36. If a man walks $8\frac{1}{2}$ miles in $2\frac{3}{4}$ hours, how far can he walk in $5\frac{1}{2}$ hours?

37. A meter, the French unit of length, is very nearly $39\frac{3}{8}$ inches. What is the difference in length between 1 yard and 1 meter? What is the difference between $5\frac{1}{4}$ yards and $5\frac{1}{4}$ meters?

38. My annual income from money loaned at \$6 on \$100 is \$36.60. How much have I loaned?

39. A company owned a cotton factory worth \$350,000. If A and B own $\frac{2}{7}$ of it, what is the value of their share?

40. Find the difference between $40\frac{1}{2}$ and $25\frac{3}{4}$; divide the difference by the sum and determine the quotient.

41. If 6 is added to both terms of the fraction $\frac{5}{8}$, is its value increased or diminished, and how much?

42. A merchant sold $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ of his tea and had 130 pounds left. How much tea had he?

43. The cost of a live turkey weighing 14 pounds was \$2. If the waste in dressing was $\frac{3}{4}$ of the weight of the turkey, for how much per pound must I sell it to gain \$1 on the cost?

44. One half of my goods was destroyed by fire; $\frac{1}{2}$ the remainder was destroyed by water and the rest cost me \$8650. What was the entire cost of the goods?

45. A carpenter alone can build a house in 25 days, but with the assistance of his son he can build it in 15 days. In how many days can the son build it?

DECIMAL FRACTIONS.

167. 1. If anything is divided into 10 equal parts, what is each part called?

2. If $\frac{1}{10}$ of a line is divided into 10 equal parts, what part of the whole line is each part? How much is $\frac{1}{10}$ of $\frac{1}{10}$?

3. If $\frac{1}{100}$ of a line is divided into 10 equal parts, what part of the line is each part? How much is $\frac{1}{10}$ of $\frac{1}{100}$?

4. How many hundredths are equal to $\frac{1}{10}$?

5. How many thousandths are equal to $\frac{1}{100}$?

6. The divisions of anything into tenths, hundredths, thousandths, etc., are called *decimal divisions*.

168. One or more of the decimal divisions of a unit is called a **Decimal Fraction**.

The word *decimal* is derived from the Latin word *decem*, which means ten.

Decimal fractions are usually called *decimals*.

169. Since decimals have the same law of increase and decrease as integers, the *denominator* of the fraction may be indicated by the *position* of the figures.

170. The figures in the *first place* at the right of units represent *tenths*; in the *second*, *hundredths*; in the *third*, *thousandths*; in the *fourth*, *ten-thousandths*; etc.

171. A period called the **Decimal Point** is placed before the decimal.

Thus, .9 represents $\frac{9}{10}$.48 represents $\frac{48}{100}$.059 represents $\frac{59}{1000}$
.2 represents $\frac{2}{10}$.08 represents $\frac{8}{100}$.007 represents $\frac{7}{1000}$

.5 represents $\frac{5}{10}$.04 represents $\frac{4}{100}$.006 represents $\frac{6}{1000}$
 .8 represents $\frac{8}{10}$ 1.09 represents $1\frac{9}{100}$.029 represents $\frac{29}{1000}$
 1.1 represents $1\frac{1}{10}$ 1.12 represents $1\frac{12}{100}$ 5.005 represents $5\frac{5}{1000}$

NUMERATION TABLE.

Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
7	9	6	.	5	8	4	6	9	4
INTEGERS.				DECIMALS.					

The orders below millionths in their order are: ten-millionths, hundred-millionths, billionths, ten-billionths, hundred-billionths, etc.

172. What order of decimals occupies

1st place ?

2d place ?

7th place ?

3d place ?

4th place ?

2d place ?

5th place ?

6th place ?

4th place ?

What decimal place is occupied by

Tenths ?

Ten-thousandths ?

Ten-millionths ?

Thousandths ?

Millionths ?

Hundredths ?

Hundredths ?

Hundred-thousandths ?

Thousandths ?

EXERCISES IN NUMERATION.**173.** 1. Read 42.356.

EXPLANATION. — The decimal part of the number expresses 3 tenths, 5 hundredths, 6 thousandths, or 356 thousandths. The whole expression is, therefore, read: 42 and 356 thousandths.

174. Hence it is evident that:

In reading a decimal, the decimal should be read as an integer, and the denomination of the right-hand figure should be added.

In reading an integer and decimal, use the word *and* only between the integral and decimal parts of the number.

Read the following :

2. .31	9. 2.63	16. 217.305
3. .73	10. 3.10	17. 301.1031
4. .281	11. 5.031	18. 130.2005
5. .032	12. 7.003	19. 800.80061
6. .0578	13. 72.070	20. 256.0032
7. .5601	14. 26.303	21. 250.00071
8. .5013	15. 31.120	22. 504.03062

EXERCISES IN NOTATION.

175. 1. Express decimally 54 thousandths.

EXPLANATION. — Since 54 thousandths is equal to 5 hundredths and 4 thousandths, 4 is written in the thousandths' place, 5 in hundredths' place, and since there are no tenths, 0 in tenths' place. A decimal point is then placed before the tenths. Hence 54 thousandths = .054.

176. Hence it is evident that :

In expressing a fraction decimally, the numerator of the fraction is written and ciphers are prefixed, if necessary, to indicate the denominator. The decimal point is then placed before tenths.

Express the following as decimals :

2. Six tenths. Four tenths. Nine tenths.
3. Thirteen hundredths. Thirty hundredths.
4. Fifteen thousandths. Two hundred one thousandths.
5. Seventeen thousandths. Six ten-thousandths.
6. Fourteen and three hundred thirty-two hundred-thousandths. Five millionths.
7. Fifty-one and seven hundred forty-two millionths.
8. Fifty and seventy-five ten-thousandths.
9. Thirty-one and sixty-four hundred-thousandths.
10. Seventy-nine and nine ten-millionths.
11. Three hundred and three millionths.
12. Sixty and sixty-seven millionths.
13. 312 millionths. 201 ten-millionths.

14. 705 ten-millionths. 605 and 703 billionths.
 15. 631 ten-billionths. 25 and 25 ten-millionths.
 16. $\frac{280}{10000}$. 19. $\frac{841}{10000}$. 22. $23710\frac{8}{1000}$.
 17. $\frac{272}{100000}$. 20. $210\frac{287}{100000}$. 23. $6751\frac{75}{10000}$.
 18. $6\frac{6}{10000}$. 21. $601\frac{51}{100000}$. 24. $3002\frac{250}{1000000}$.

REDUCTION OF DECIMALS.

177. To reduce dissimilar decimals to similar decimals.

1. How do the fractions $\frac{6}{10}$, $\frac{60}{100}$, and $\frac{600}{1000}$ compare in value? How do the decimals expressing them compare in form? What is the effect of annexing a cipher to a decimal?

2. How do the fractions $\frac{6}{10}$, $\frac{6}{100}$, and $\frac{6}{1000}$ compare in value? How do the decimals expressing them compare in form? What is the effect of prefixing a cipher to a decimal?

3. How does the number of places in a decimal compare with the number of ciphers in the denominator?

178. Hence it is evident that:

1. *Annexing ciphers to a decimal does not alter its value.*
2. *Each decimal cipher prefixed to a decimal diminishes the value of the decimal tenfold.*
3. *The number of places in a decimal is equal to the number of ciphers in the denominator of its equivalent common fraction.*

WRITTEN EXERCISES.

179. 1. Reduce .6, .46, and .095 to similar fractions.

EXPLANATION. — Since the lowest order of the given decimals is thousandths, all the decimals must be changed to thousandths. Since annexing ciphers to a decimal does not alter its value, we give to each number three decimal places by annexing ciphers, thus making them similar by reducing them all to thousandths.

.6 = .600
 .46 = .460
 .095 = .095

2. Reduce to similar decimals: .0321, .00623, .00025, .75.
3. Reduce to ten-thousandths: .68, .0785, .063, .251, .27.
4. Express 8 tenths as hundredths; as thousandths.
5. Change to similar decimals: .003, .02, .2100, .6530.

180. To reduce a decimal to a common fraction.

WRITTEN EXERCISES.

1. Reduce .75 to its equivalent common fraction.

EXPLANATION. — .75 expressed as a common fraction is $\frac{75}{100} = \frac{3}{4}$, which, being reduced to its smallest terms, is $\frac{3}{4}$.

181. From the explanation it is apparent that:

A decimal may be reduced to a common fraction by omitting the decimal point and supplying the proper denominator.

The result should be expressed in its lowest terms.

Reduce the following to common fractions in their smallest terms :

- | | | | |
|---------|----------|-------------|-------------|
| 2. .75. | 6. .125. | 10. .0045. | 14. .00375. |
| 3. .50. | 7. .875. | 11. .075. | 15. .00625. |
| 4. .35. | 8. .625. | 12. .00875. | 16. .05375. |
| 5. .80. | 9. .375. | 13. .0250. | 17. .0875. |

182. To reduce a common fraction to a decimal.

1. How many tenths are there in $\frac{1}{2}$? In $\frac{1}{3}$?
2. How many hundredths are there in $\frac{1}{4}$? In $\frac{1}{5}$? In $\frac{3}{4}$?
3. How many hundredths are there in 1? In $\frac{1}{2}$? In $\frac{3}{8}$?
4. How many thousandths are there in 1? In $\frac{1}{4}$? In $\frac{3}{8}$?

WRITTEN EXERCISES.

183. 1. Reduce $\frac{5}{8}$ to an equivalent decimal.

EXPLANATION. — $\frac{5}{8}$ is $\frac{1}{8}$ of 5, or of 50 tenths; and $\frac{1}{8}$ of 50 tenths is 6 tenths, and 2 tenths remaining. 2 tenths are equal to 20 hundredths, and $\frac{1}{8}$ of 20 hundredths is 2 hundredths and 4 hundredths remaining. 4 hundredths are equal to 40 thousandths, and $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Hence $\frac{5}{8}$ is equal to 6 tenths + 2 hundredths + 5 thousandths, or .625.

184. From the process it is evident that:

A common fraction may be reduced to a decimal by annexing ciphers to the numerator and dividing by the denominator, and pointing off as many decimal places in the quotient as there are ciphers annexed.

In many cases the division is not exact. In such instances the remainder may be expressed as a common fraction, or the sign + may be employed after the decimal to show that the result is not complete; thus $\frac{1}{3} = .166\frac{2}{3}$, or $.166+$.

Reduce the following to decimals:

- | | | | | | |
|---------------------|---------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 2. $\frac{3}{10}$. | 6. $\frac{3}{75}$. | 10. $\frac{19}{80}$. | 14. $\frac{3}{7}$. | 18. $1\frac{1}{2}$. | 22. $\frac{4}{5}$. |
| 3. $\frac{5}{8}$. | 7. $\frac{3}{20}$. | 11. $\frac{27}{88}$. | 15. $1\frac{5}{11}$. | 19. $8\frac{2}{3}$. | 23. $\frac{5}{32}$. |
| 4. $\frac{3}{16}$. | 8. $\frac{1}{40}$. | 12. $\frac{5}{11}$. | 16. $\frac{4}{825}$. | 20. $3\frac{1}{8}$. | 24. $\frac{3}{80}$. |
| 5. $\frac{5}{20}$. | 9. $\frac{2}{38}$. | 13. $\frac{4}{87}$. | 17. $3\frac{1}{2}$. | 21. $6\frac{7}{10}$. | 25. $\frac{3}{500}$. |

ADDITION OF DECIMALS.

- 185.** 1. What is the sum of $\frac{3}{10}$ and $\frac{4}{10}$? Of .4 and .7?
 2. What is the sum of $\frac{5}{100}$ and $\frac{12}{100}$? Of .08 and .09?
 3. What is the sum of $\frac{1}{10}$ and $\frac{1}{100}$? Of .5 and .25?

WRITTEN EXERCISES.

- 186.** 1. What is the sum of 2.85, 1.625, and 3.0235?

2.8500	or 2.85	EXPLANATION. — The numbers are written so that units stand under units, tenths under tenths, etc. The decimals may be made similar by annexing ciphers and then added, care being taken to place the decimal point in the sum directly under the decimal point in the numbers added. Or, since only units of one order are found in any column, it is not necessary, in practice, to make the decimals similar.
1.6250	1.625	
3.0235	3.0235	
<hr/> 7.4985	<hr/> 7.4985	

Find the sum of the following:

- | | |
|------------------------------|------------------------------|
| 2. $8.75 + .369 + 75.212$. | 4. $3.801 + 2.875 + 1.625$. |
| 3. $1.89 + 62.625 + 3.939$. | 5. $1.25 + 4.275 + 48.35$. |

6. $.356 + 1.275 + 32.0325$. 11. $3.495 + 1.87 + 3.25$.
 7. $15.815 + 17.21 + 31.41$. 12. $38.835 + 8.9764 + 1.385$.
 8. $.2735 + 1.596 + 3.25$. 13. $1.8035 + 3.00412 + 31.025$.
 9. $2.948 + 3.7642 + 85.035$. 14. $62.589 + 3.875 + 1.3759$.
 10. $46.2 + 381.25 + 2.985$. 15. $28.48 + 17.578 + 34.752$.

16. What is the sum of 8 tenths, 12 hundredths, 75 thousandths, 69 ten-thousandths, 21 hundredths?

17. What is the sum of 312 ten-thousandths, 8 millionths, 65 thousandths, 413 ten-millionths, 9 tenths?

18. What is the sum of 82 hundredths, 510 thousandths, 6 tenths, 80 ten-thousandths?

19. What is the sum of 303 thousandths, 4108 millionths, 635 ten-thousandths, 803 ten-millionths?

20. A bicyclist traveled on his wheel 51.875 miles in one day, 45.75 the next, and 35.375 the next day. How far did he travel during these three days?

21. A ticket agent sold in one month tickets to the value of \$18,756.75, in the next \$17,983.25, in another \$2001.875. How much were his sales during those three months?

SUBTRACTION OF DECIMALS.

187. 1. From $\frac{8}{10}$ take $\frac{5}{10}$. From .18 take .13.
 2. From $\frac{7}{100}$ take $\frac{5}{100}$. From .035 take .026.
 3. From 4 take $\frac{5}{10}$. From 8 take .8.

WRITTEN EXERCISES.

188. 1. From 46.57 subtract 9.46325.

$$46.57000 = 46.57$$

$$9.46325 = 9.46325$$

$$37.10675 \quad 37.10675$$

EXPLANATION. — The numbers are written so that units stand under units, tenths under tenths, etc. The decimals may be made similar and then subtracted, care being taken to place the decimal point in the remainder directly under the decimal point in the number subtracted. The ciphers may be supposed to be annexed when we subtract, and consequently need not be written.

	2.	3.	4.	5.	6.
From	25.38	39.85	87.35	46.81	24.37
Take	<u>14.05</u>	<u>29.755</u>	<u>59.355</u>	<u>21.35</u>	<u>12.358</u>
	7.	8.	9.	10.	11.
From	72.189	78.895	26.825	34.24	21.12
Take	<u>35.976</u>	<u>53.5987</u>	<u>21.3452</u>	<u>30.16</u>	<u>12.31</u>

12. What is the difference between 6.325 and 1.0345?
13. What is the difference between 6.45 and 2.3375?
14. From 65 hundredths subtract 65 thousandths.
15. From 162 thousandths subtract 261 millionths.
16. Find the value of $23.2 - .0325 + 6.95 - 3.4876$.
17. Find the value of $6.594 - 2.0341 - .70325 + 8.963$.
18. Find the value of $81.8 + 35.625 - 38.875 - 2.0034$.
19. Find the value of $4.6 + 2.635 - .8925 - .0359$.
20. From a cistern that contained 30.5 barrels of water, 25.75 barrels were drawn off. How much water remains in the cistern?
21. A man bought a tub of butter for \$35.75, paying \$10 in cash and the rest in groceries. How much did he pay in groceries?
22. A man whose income was \$10,000 spent in one year \$7375.87. How much did he save that year?

MULTIPLICATION OF DECIMALS.

189. 1. What is the product of $\frac{1}{10} \times \frac{1}{10}$? Of $.1 \times .1$? Of $.5 \times .5$? How does the number of ciphers in the denominator of the product of these fractions compare with the number of ciphers in the denominators of the factors?

2. What is the product of $\frac{1}{10} \times \frac{1}{100}$? Of $.1 \times .01$? Of $.5 \times .05$? How does the number of ciphers in the denominator of the product of these fractions compare with the number of ciphers in the denominators of the factors?

3. How does the number of decimal places in a decimal fraction compare with the number of ciphers in its denominator?

4. How, then, does the number of decimal places in the product of two or more decimals compare with the number of places in the factors?

190. It is apparent, therefore, that:

The number of decimal places in the product of two or more decimals is equal to the number of decimal places in the factors.

WRITTEN EXERCISES.

191. 1. What is the product of .275 multiplied by .17?

$\begin{array}{r} .275 \\ .17 \\ \hline 1925 \\ 275 \\ \hline .04675 \end{array}$	<p>EXPLANATION. — Since 275 and 17 are the numerators of the fractions, in multiplying the fractions, we must multiply 275 by 17, which gives 4675 for a new numerator.</p> <p>Since thousandths multiplied by hundredths produce hundred-thousandths, the product of the fractions is 4675 hundred-thousandths, or .04675.</p> <p>Or the number of decimal places in the product will be five, the number of decimal places in both factors. Art. 222.</p>
---	---

192. Hence, it is evident that:

To multiply decimals we multiply as if the numbers were integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.

If the product does not contain a sufficient number of decimal places, the deficiency must be supplied by *prefixing* ciphers.

Multiply:

- | | | |
|----------------|-----------------|-------------------|
| 2. .25 by .75. | 5. .75 by 2.06. | 8. .876 by .375. |
| 3. .24 by 3.4. | 6. .93 by .047. | 9. .433 by 2.75. |
| 4. .57 by 68. | 7. .39 by .875. | 10. 72.2 by .055. |

11. 3.62 by .0037. 15. 2.37 by 7.563. 19. .1895 by .0436.
 12. 1.98 by 2.034. 16. 2.53 by .00635. 20. 6.81 by 0.0875.
 13. 25.7 by 3.887. 17. .3147 by .0052. 21. 578 by .0934.
 14. 15.8 by .0855. 18. .1963 by .00638. 22. 0.765 by .067.

23. Multiply 5.372 by 100.

$$\begin{array}{r} 5.372 \\ \times 100 \\ \hline 537.200 \end{array}$$

EXPLANATION. — Since each removal of a figure one place to the left increases its value tenfold, the removal of the decimal point one place to the right multiplies by 10, and its removal two places to the right multiplies by 100; or, the process may be performed in the ordinary manner.

24. Multiply 27.65 by 10. 28. Multiply 6.8735 by 100.
 25. Multiply 49.675 by 10. 29. Multiply .3768 by 1000.
 26. Multiply 2.795 by 100. 30. Multiply 6.835 by 100.
 27. Multiply 8.935 by 100. 31. Multiply .7634 by 1000.

32. Multiply 89.92 by 10. By 100. By 1000. By 10,000.

33. Multiply 68.3025 by 100. By 1000. By 10,000.

34. Which is larger and how much, the product of .625 and .375, or their sum?

35. What will 52.75 bushels of oats cost at \$.45 per bushel?

36. A merchant sold 24.25 yards of muslin at \$.125 a yard. How much did he receive for it?

37. A man bought 214 bags of barley. What did they cost him, if each bag held 2.5 bushels, and the price was \$.96 a bushel?

38. A tailor bought 4.5 yards of cassimere at \$2.25 a yard, and 6.25 yards of broadcloth at \$6.50 a yard. How much more did the broadcloth cost than the cassimere?

39. How many acres are left in a farm of 563.27 acres, if 3 fields of 57.135 acres each are sold from it?

40. A steamship in crossing the Atlantic sailed at an average speed of 325.75 miles per day. If another steamer sailed from the same port at the same time at the rate of 395.35 miles per day, how far were they apart in 5.75 days?

DIVISION OF DECIMALS.

193. 1. When a number is divided by 3, what part of it is found? By 6? By 8? By 15?

2. How much is $\frac{1}{3}$ of 9? $9 \div 3$? $\frac{1}{3}$ of 15? $15 \div 5$?

3. How much is $\frac{1}{3}$ of 9 tenths? $\frac{1}{4}$ of .8? $\frac{1}{5}$ of .6? $\frac{1}{2}$ of .6?

4. How much is $\frac{1}{3}$ of 25 hundredths? $\frac{1}{3}$ of .24? $\frac{1}{3}$ of .36?

WRITTEN EXERCISES.

194. 1. Find 1 sixth of 109.92, or divide 109.92 by 6.

$$\begin{array}{r} 6 \overline{)109.92} \\ 18.32 \end{array}$$

EXPLANATION.—One sixth of 10 tens is 1 ten and 4 tens remaining; 1 sixth of 4 tens remaining and 9 units, or 49 units, is 8 units and 1 unit remaining; 1 sixth of 1 unit remaining and 9 tenths, or 19 tenths, is 3 tenths and 1 tenth remaining; 1 sixth of 1 tenth remaining and 2 hundredths, or 12 hundredths, is 2 hundredths. Therefore, one sixth of 109.92 is 18.32, or the quotient of $109.92 \div 6$ is 18.32.

2. What is 1 fifth of 86.4055? 5. $\$234.54 \div 9 = ?$

3. What is 1 eighth of 94076.8? 6. $\$907.34 \div 7 = ?$

4. What is 1 eleventh of 1086.415? 7. $\$324.784 \div 8 = ?$

8. Find 1 eighth of 9675 acres.

$$\begin{array}{r} 8 \overline{)9675.000} \\ 1209.375 \end{array}$$

EXPLANATION.—We divide as in simple division until we come to the remainder, 3 acres. This we change to 30 tenths and continue the division thus:

1 eighth of 30 tenths is 3 tenths and 6 tenths, or 60 hundredths, remainder. 1 eighth of 60 hundredths is 7 hundredths and 4 hundredths, or 40 thousandths, remainder; 1 eighth of 40 thousandths is 5 thousandths. Therefore, 1 eighth of 9675 acres is 1209.375 acres.

How much is:

9. $\frac{1}{4}$ of \$23.50? 12. $\frac{1}{4}$ of 392.25? 15. $8.643 \div 6$?

10. $\frac{1}{3}$ of \$36.75? 13. $\frac{1}{3}$ of 468.36? 16. $85.86 \div 9$?

11. $\frac{1}{8}$ of \$59.64? 14. $\frac{1}{8}$ of 693.34? 17. $3.845 \div 5$?

18. A man received \$3278 for 5 acres of land. What was the price per acre?

19. An estate valued at \$35,267 was divided equally among 8 children. What was each child's share?

195. The number of decimal places in the quotient.

1. What is the product of $.5 \times .7$? How many decimal places does it contain? If $.35$ is divided by $.7$, what will the quotient be? How many decimal places does it contain?

2. What is the product of $.9 \times .05$? If $.045$, the product, is divided by $.9$, one of the factors, what is the other factor? How many decimal places does it contain?

3. Since the product contains as many decimal places as there are in all the factors, if the product of two factors is given and one of the factors, how may the number of decimal places in the other factor be found?

4. Since the dividend may be regarded as the product of two factors, one of which is the divisor, and the other the quotient, how may the number of decimal places in the quotient be found from those in the dividend and divisor?

196. It is therefore evident that:

The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.

WRITTEN EXERCISES.**197. 1. Divide $.15652$ by $.043$.****.043).15652(3.64**

$$\begin{array}{r}
 129 \\
 \underline{275} \\
 258 \\
 \underline{172} \\
 172 \\
 \underline{}
 \end{array}$$

EXPLANATION.—Since the dividend is equal to the product of the divisor and quotient, if 15652 , the numerator of the dividend, is divided by 43 , the numerator of the divisor, the result obtained will be the numerator of the quotient. Dividing, it is found to be 364 . Hence, the numerator of the quotient is found precisely as in simple division. The denominator of the quotient may be found by dividing the denominator of the dividend by the denominator of the divisor, that is by dividing 100000 by 1000 , which gives 100 . Therefore, the quotient is $\frac{364}{100}$, or 3.64 . Or,

The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor. Art. 228.

198. Hence it is evident that:

In dividing decimals, we divide as if the numbers were integers, and from the right of the quotient point off as many places for decimals as the number of decimal places in the dividend exceeds the number in the divisor.

1. If the quotient does not contain a sufficient number of decimal places, the deficiency must be supplied by *prefixing* ciphers.

2. Before commencing the division, the number of decimal places in the dividend *should be made at least equal* to the number of decimal places in the divisor.

3. When there is a remainder after using all the figures of the dividend, annex decimal ciphers and continue the division.

4. For business purposes, it is not necessary to carry the division further than to four or five decimal figures in the quotient.

Divide:

- | | |
|------------------------|----------------------|
| 2. 78.12 by 3.6. | 17. 14.91 by 70. |
| 3. 12.32 by 3.12. | 18. 8.25 by 1.5. |
| 4. 1675.8 by .49. | 19. .00864 by .24. |
| 5. 11.904 by 3.1 | 20. 107.5 by .043. |
| 6. 452.92 by .052. | 21. .0009 by .125. |
| 7. 63.168 by .94. | 22. .07245 by .23. |
| 8. .15275 by .325. | 23. 800 by .0004. |
| 9. .12376 by .0364. | 24. 12.126 by .235. |
| 10. .0184428 by .5123. | 25. 167.544 by 71.6. |
| 11. .29184 by .0057. | 26. 56.112 by 0.56. |
| 12. 4.5872 by .0122. | 27. 2.1828 by .034. |
| 13. .056448 by 15.68. | 28. 4066.2 by .648. |
| 14. .0816785 by 6.235. | 29. .08 by 1.611. |
| 15. .0983253 by .0673. | 30. 114.87 by .0035. |
| 16. 5.97066 by 35.73. | 31. 30000 by .00003. |

32. Divide 325.47 by 100.

100)325.47 **EXPLANATION.** — Since each removal of a figure one place to the right decreases its value tenfold, the removal of the decimal point one place to the left divides by 10, and the removal two places by 100.

199. Hence it is evident that :

To divide by 1 with any number of ciphers annexed, the decimal point should be removed as many places to the left as there are ciphers annexed to the 1.

- | | |
|---------------------------|----------------------------|
| 33. Divide 392.5 by 100. | 36. Divide 4.825 by 100. |
| 34. Divide 26.45 by 10. | 37. Divide 38.62 by 1000. |
| 35. Divide 369.5 by 1000. | 38. Divide 42.85 by 10000. |

39. How many yards of cloth, at \$ 5.75 per yard, can be bought for \$ 153.525 ?

40. How many barrels of apples, at \$ 1.25 per barrel, can be bought for \$ 18.43 $\frac{1}{4}$?

41. How many cords of wood, at \$ 5.62 $\frac{1}{2}$ per cord, can be bought for \$ 1495.12 $\frac{1}{2}$?

ACCOUNTS AND BILLS.

200. A record of the debts and credits between two parties is called an **Account**.

201. A statement of the quantity and price of the articles sold, and the value of the whole, is called a **Bill**.

A bill is *receipted* when the words *Received Payment*, or *Paid*, are written at the bottom, and the seller's name is signed either by himself or by some authorized person.

202. The following abbreviations are in common use :

@,	At.	Do.,	The same.	Mdse.,	Merchandise.
%,	Account.	Doz.,	Dozen.	No.,	Number.
Acc't,	Account.	Dr.,	Debtor.	Pay't,	Payment.
Bal.,	Balance.	Fr't,	Freight.	Pd.,	Paid.
Bbl.,	Barrel.	Hhd.,	Hogshead.	Per,	By.
Bo't,	Bought.	Inst.,	This month.	Rec'd,	Received.
Co.,	Company.	Int.,	Interest.	Yd.,	Yard.
Cr.,	Creditor.	Lb.,	Pound.	Yr.,	Year.

1.

*Brooklyn, N.Y., June 25, 1892.**Mr. Horace E. Gray,*

Bought of JONES & PURDY.

4 bags Coffee,	300 lb.	@	\$.35½	\$ 106	50
3 chests Tea,	260 "	"	.87½	227	50
8 boxes Raisins,	200 "	"	.12½	25	00
2 bbl. Coffee-sugar,	432 "	"	.06½	27	00
2 bbl. Crackers,	150 "	"	.05½	8	25
Received payment,				\$ 394	25
<i>Jones & Purdy,</i>					
<i>Per G. I.</i>					

Make out in proper form, find the footings of the following bills and accounts, and receipt them :

2. Miss Lucy Graham bought of Luckey, Platt & Co. 15 yards of calico @ 10 cents; 36 yards of sheeting @ 18½ cents; 2 pairs of gloves @ \$1.50; 1 sun-umbrella @ \$3.75; 5 yards of Hamburg edging @ 25 cents; 6 handkerchiefs @ 37½ cents; and 7 pairs of hose @ 50 cents.

3. Messrs. Henry Davis & Co. bought of James Harkness 150 barrels of flour @ \$5.90; 105 bushels of wheat @ \$1.20; 325 bushels of corn @ \$.68; 675 bushels of oats @ \$.40; and 50 barrels of potatoes @ \$1.25.

4. Mrs. Charles Reid in account with Roger Caldwell: August 7, 1892, 9 yards cashmere @ \$.75; ¼ yard velvet @ \$1.50; 12 yards lawn @ .12½; 1½ yards silesia @ \$.30; ¾ yards silk at \$1.50.

5. Darwin Howard bought of Stone & Bacon 15 pounds nails @ 4½¢; 4 pairs of hinges @ 37½¢; 18 dozen screws @ 11¢; 5 dozen milk-pans @ \$2.75; 7 locks @ 75¢; 2 saws @ \$1.35; 3 hammers @ 85¢; and 2 planes @ \$1.50.

REVIEW.

WRITTEN EXERCISES.

203. 1. How much will 2 dozen pairs of kid gloves cost at \$1.25 a pair?

2. A man paid out the following sums: For a pair of horses, \$375; for a carriage, \$295; for harnesses, \$115. How much did he pay for all?

3. A farmer sold 12.35 cords of wood at \$4.75 per cord. How much did he receive for it?

4. A man purchased 8 tons of coal at \$5.75 per ton, and 7 cords of hard wood at \$5.50 per cord. How much did he pay for fuel?

5. If illuminating gas is sold at \$2.50 per thousand cubic feet, for how much gas will \$17.50 pay?

6. Find the sum of 15 dollars 9 cents 3 mills, 12 dollars 5 cents 4 mills, 18 dollars 14 cents 7 mills, and 16 dollars 10 cents.

7. If 5.5 yards of ribbon cost \$2.75, what will 9.75 yards cost?

8. If a clerk earns \$520 per year and spends \$110 for clothes, \$215 for board, and \$75.50 for other expenses, how long will it take him to save \$1000?

9. If 54,600 letters were mailed from a post-office in 30 days, what was the average number mailed per day?

10. How many pairs of curtains can be bought for \$414, if 12 pairs are bought for \$138?

11. A drover bought 280 head of cattle for \$12,740, and sold them at a gain of \$8.75 per head. How much did he receive for them per head?

12. A man paid \$87.50 for materials to paint his house, and he paid 3 painters \$2.50 per day for 4.5 days to do the work. How much was the entire expense of painting his house?

13. The distance between two places is $168\frac{3}{4}$ miles. How long will it take a bicyclist to ride the distance at 12.5 miles per hour?

14. How much will one dozen each of cups and saucers cost, if the cups cost \$.74 $\frac{3}{4}$ and the saucers \$.67 $\frac{1}{2}$ apiece?

15. What must I pay for 3 tables at \$12.75 each, 2 chairs at \$9.50 each, 6 chairs at \$3.75 each, and one sofa at \$25.50?

16. A merchant sold 35.5 yards of cambric at 25 cents per yard and gained \$1.775. How much did it cost him per yard?

17. During the Civil War a dollar bill was, at one time, worth only 65 cents in gold. How many dollars in gold were 520 dollars in paper money worth at that time?

18. A farmer received for his barley \$151.37, for his oats \$196.84, for his corn as much as for barley and oats, for his rye \$75.93, and for his wheat as much as for all the others. For how much did he sell his grain?

19. A dealer bought 25 barrels of flour, each containing 196 pounds, for \$112.50. How much did it cost him per pound? How much will be his entire gain if he sells it at 3 $\frac{1}{2}$ cents per pound?

20. A merchant paid the following sums, on the average, for expenses during each month: For rent, \$275; for heating and lighting, \$115; for clerk hire, \$375; and for other expenses, \$85. What will be his net gain annually, if his gross profits are \$12,560?

21. An ice-dealer put into his ice-house 1315 tons of ice. He sold at different times the following quantities: 216.75 tons, 396.5 tons, 542.7 tons. If the rest melted during the summer, how many tons melted, and how many tons did he sell?

22. If a freight train runs 15.75 miles per hour, in how many hours will it run 189 miles?

DENOMINATE NUMBERS.

204. A number used in connection with some specified thing is called a **Concrete Number**.

Thus, 5 days, 6 men, 4 feet, 8 pounds are concrete numbers.

205. A number used without reference to any particular thing is called an **Abstract Number**.

Thus, 5, 6, 4, 8, are abstract numbers.

206. A concrete number in which the unit of measure is established by law or custom is called a **Denominate Number**.

Thus, 5 gallons, 8 rods, 5 acres, are denominate numbers.

207. A denominate number which is composed of units of one denomination only is called a **Simple Denominate Number**.

Thus, 3 feet, 8 ounces, 5 quarts, are simple denominate numbers.

208. A denominate number which is composed of units of two or more denominations that are related to each other is called a **Compound Denominate Number**.

Thus, 5 yards 2 feet 8 inches is a compound denominate number.

MEASURES OF LENGTH.

209. In measuring length or distance, Measures of Length or Linear Measures are used.

TABLE.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
5½ yards }	= 1 rod (rd.).
16½ feet }	
320 rods	= 1 mile (mi.).

EXERCISES.

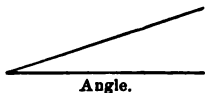
1. How many in. are there in 2 ft. ? In 3 ft. ? In 4 ft. ?
2. How many ft. are there in 4 yd. ? In 5 yd. ?
3. How many yd. are there in 4 rd. ? In 6 rd. ? In 8 rd. ?
4. How many ft. are there in 2 rd. ? In 5 rd. ? In 10 rd. ?
5. How many rd. are there in 2 mi. ? In 3 mi. ?
6. How many ft. are there in 24 in. ? In 36 in. ?
7. How many yd. are there in 9 ft. ? In 15 ft. ? In 24 ft. ?
8. How many rd. are there in 33 ft. ? In 66 ft. ?
9. How many mi. are there in 640 rd. ? In 960 rd. ?
10. How many in. are there in $\frac{1}{2}$ ft. ? In $\frac{2}{3}$ ft. ? In $\frac{3}{4}$ ft. ?
11. How many ft. are there in $\frac{1}{8}$ yd. ? In $1\frac{1}{8}$ yd. ?
12. How many in. are there in 2 ft. ? In 2 ft. 6 in. ?
13. How many ft. are there in 1 yd. ? In 1 yd. 2 ft. ? In 2 yd. ? In 2 yd. 1 ft. ? In 3 yd. 2 ft. ?
14. How many in. are there in 1 yd. ? In 1 yd. 1 ft. ?

MEASURES OF SURFACE.

210. Anything that has only length and breadth is called a Surface.

Thus, this page, the side of a block, etc., are surfaces.

211. The difference in the direction of two lines that meet is called an Angle.



212. A figure that has four equal sides and four equal angles is called a Square.

1. A *square inch* is a square each of whose sides is an *inch* long ; a *square foot* is a square each of whose sides is a *foot* long.

2. The angles of a square are called *right angles*.

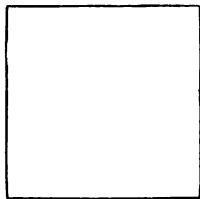


TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
30 $\frac{1}{2}$ square yards } 272 $\frac{1}{2}$ square feet }	= 1 square rod (sq. rd.).
160 square rods	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

EXERCISES.

1. How many sq. in. are there in 2 sq. ft.? In 3 sq. ft.?
2. How many sq. ft. are there in 3 sq. yd.? In 4 sq. yd.?
3. How many sq. rd. are there in 2 A.? In 3 A.?
4. How many sq. yd. are there in 18 sq. ft.? In 36 sq. ft.?
5. How many sq. ft. are there in 288 sq. in.? In 432 sq. in.?
6. How many A. are there in $\frac{1}{2}$ sq. mi.? In $\frac{1}{4}$ sq. mi.?
7. How many sq. rd. are there in $\frac{1}{2}$ A.? In $\frac{1}{4}$ A.?
8. How many sq. ft. are there in $\frac{1}{8}$ sq. yd.? In $\frac{3}{8}$ sq. yd.?
9. How many sq. in. are there in $\frac{1}{2}$ sq. ft.? In $\frac{1}{8}$ sq. ft.?
10. How many sq. ft. are there in 2 sq. ft. 12 sq. in.?
In 5 sq. ft. 80 sq. in.?

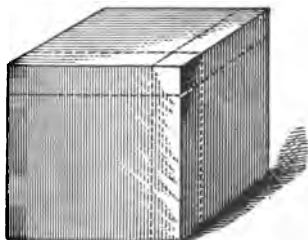
MEASURES OF VOLUME.

213. Anything that has length, breadth, and thickness is called a **Solid**.

214. The number of solid units in a body is called its *volume* or *solid contents*.

215. A solid having six equal square sides, called faces, is a **Cube**.

A solid whose faces are each a square inch is a *cubic inch*; a solid whose faces are each a square foot is a *cubic foot*.



Cube.

TABLE.

1728 cubic inches (cu. in.)	= 1 cu. ft. (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).
128 cubic feet	= 1 cord of wood or stone (C.).

EXERCISES.

1. How many cu. in. are there in 2 cu. ft.? In 5 cu. ft.?
2. How many cu. ft. are there in 4 cu. yd.? In 8 cu. yd.?
3. How many cu. ft. are there in 2 C.? In 3 C.?
4. How many cu. yd. are there in 54 cu. ft.? In 81 cu. ft.?
5. How many cu. in. are there in $\frac{1}{2}$ cu. ft.? In $\frac{1}{4}$ cu. ft.?
6. How many cu. ft. are there in $\frac{1}{4}$ C.? In $\frac{3}{8}$ C.?
7. How many cu. ft. are there in 2 cu. yd. 3 cu. ft.?
8. How many cu. in. are there in 1 cu. ft. 72 cu. in.?

LIQUID MEASURES.

216. Measures used in measuring all fluids and liquids are called **Liquid Measures.**

TABLE.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).

1. In determining the capacity of cisterns, reservoirs, etc., $31\frac{1}{2}$ gallons are considered a barrel (bbl.), and 2 barrels, or 63 gallons, a hogshead (hhd.).
2. A gallon contains 231 cubic inches.

EXERCISES.

1. How many gi. are there in 2 pt.? In 3 pt.? In 5 pt.?
2. How many pt. are there in 3 qt.? In 5 qt.? In 8 qt.?
3. How many qt. are there in 5 gal.? In 6 gal.?
4. How many gal. are there in 24 qt.? In 40 qt.?
5. How many qt. are there in 16 pt.? In 20 pt.?
6. How many gal. are there in 16 pt.? In 20 pt.?

7. How many pt. are there in $\frac{1}{2}$ gal. ? In $\frac{1}{4}$ gal. ?
8. How many pt. are there in 3 qt. ? In 3 qt. 1 pt. ?
9. How many gi. are there in 1 pt. 3 gi. ? In 1 pt. 2 gi. ?
10. How many pt. are there in 1 gal. ? In 2 gal. 3 qt. ?

DRY MEASURES.

217. Measures used in measuring fruit, roots, vegetables, and grain are termed **Dry Measures**.

TABLE.

2 pints (pt.)	= 1 (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

1. A bushel contains 2150.4 cubic inches.
2. A quart and a pint dry measure contain a little more than the same liquid measures.

EXERCISES.

1. How many pt. are there in 5 qt. ? In 7 qt. ?
2. How many qt. are there in 5 pk. ? In 4 pk. ?
3. How many pk. are there in 3 bu. ? In 6 bu. ?
4. How many bu. are there in 24 pk. ? In 36 pk. ?
5. How many pk. are there in 40 qt. ? In 32 qt. ?
6. How many qt. are there in 16 pt. ? In 20 pt. ?
7. How many pk. are there in $\frac{1}{2}$ bu. ? In $\frac{1}{4}$ bu. ?
8. How many pk. are there in 2 bu. 3 pk. ?

AVOIRDUPOIS WEIGHT.

218. In weighing groceries and all heavy articles as hay, grain, coal, including all metals except gold and silver, **Avoirdupois Weight** is used.

TABLE.

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundred-weight (cwt.).
20 hundred-weight } 2000 pounds	= 1 ton (T.).

EXERCISES.

1. How many oz. are there in 2 lb.? In 3 lb.? In $4\frac{1}{2}$ lb.?
2. How many lb. are there in 4 cwt.? In 6 cwt.?
3. How many lb. are there in 3 T.? In 5 T.? In $6\frac{1}{4}$ T.?
4. How many cwt. are there in 300 lb.? In 400 lb.?
5. How many T. are there in 2500 lb.? In 3000 lb.?
In 5000 lb.?
6. How many lb. are there in $\frac{1}{2}$ T.? In $\frac{1}{4}$ T.? In $\frac{3}{4}$ T.?
7. How many oz. are there in $\frac{1}{10}$ T.? In $\frac{7}{100}$ T.?

TROY WEIGHT.

219. In weighing gold, silver, and jewels **Troy Weight** is used.

TABLE.

24 grains (gr.) = 1 pennyweight (pwt.).

20 pennyweights = 1 ounce (oz.).

12 ounces = 1 pound (lb.).

The Troy lb. contains 5760 gr.; the Avoirdupois lb., 7000 gr.

The Troy oz. contains 480 gr.; the Avoirdupois oz., $437\frac{1}{2}$ gr.

EXERCISES.

1. How many gr. are there in 2 pwt.? In 4 pwt.?
2. How many pwt. are there in 2 oz.? In 4 oz.? In 6 oz.?
3. How many oz. are there in 4 lb.? In $4\frac{1}{2}$ lb.? In $5\frac{1}{4}$ lb.?
4. How many gr. are there in 1 oz.? In $\frac{1}{2}$ oz.? In $\frac{1}{4}$ oz.?
5. How many lb. are there in 24 oz.? In 36 oz.?
6. How many oz. are there in 40 pwt.? In 50 pwt.?
7. How many pwt. are there in 48 gr.? In 60 gr.?
8. How many oz. are there in 1 lb.? In 2 lb. 4 oz.?

APOTHECARIES' WEIGHT.

220. In weighing medicines for prescriptions **Apothecaries' Weight** is used by apothecaries and physicians.

TABLE.

20 grains (gr.)	= 1 scruple (sc., or \odot).
3 scruples	= 1 dram (dr., or \mathfrak{z}).
8 drams	= 1 ounce (oz., or \mathfrak{z}).
12 ounces	= 1 pound (lb., or lb).

EXERCISES.

1. How many gr. are there in 3 sc.? In 4 sc.? In $5\frac{1}{2}$ sc.?
2. How many sc. are there in 4 dr.? In 5 dr.? In 10 dr.?
3. How many dr. are there in 3 oz.? In 8 oz.? In $9\frac{1}{2}$ oz.?
4. How many oz. are there in 2 lb.? In $4\frac{1}{2}$ lb.? In $5\frac{3}{4}$ lb.?
5. How many lb. are there in 48 oz.? In 60 oz.? In 66 oz.?
6. How many oz. are there in 24 dr.? In 40 dr.? In 52 dr.?
7. How many dr. are there in 12 sc.? In 18 sc.? In 22 sc.?
8. How many sc. are there in 40 gr.? In 50 gr.? In 90 gr.?
9. How many dr. are there in 2 oz. 3 dr.? In 3 oz. 5 dr.?

DIVISIONS OF TIME.

221.

TABLE.

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (da.).
7 days	= 1 week (wk.).
365 days	= 1 year (yr.).
366 days	= 1 leap year (l. yr.).
100 years	= 1 century (C.).

Centennial years whose number is exactly divisible by 400, and other years whose number is exactly divisible by 4, are *Leap Years*.

There are 12 months in a year. The names and the number of days in each are as follows :

- | | |
|---------------------------------|-----------------------------|
| 1. January (Jan.) 31 days. | 7. July (July) 31 da. |
| 2. February (Feb.) 28 or 29 da. | 8. August (Aug.) 31 da. |
| 3. March (Mar.) 31 da. | 9. September (Sept.) 30 da. |
| 4. April (Apr.) 30 da. | 10. October (Oct.) 31 da. |
| 5. May (May) 31 da. | 11. November (Nov.) 30 da. |
| 6. June (June) 30 da. | 12. December (Dec.) 31 da. |

The 29th day of February is the day added to make the leap year.

The following rhyme will serve to fix in memory the days in each month :

Thirty days hath September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight, and one day more
We add to it one year in four.

EXERCISES.

1. How many sec. are there in 5 min.? In 6 min.? In 10 min.?
2. How many min. are there in 2 hr.? In 3 hr.?
3. How many hr. are there in 2 da.? In 4 da.? In $\frac{1}{2}$ da.?
4. How many da. are there in 3 wk.? In 6 wk.?
5. How many wk. are there in 14 da.? In 35 da.?
6. How many hr. are there in 240 min.? In 180 min.?
7. How many min. are there in 120 sec.? In 300 sec.?
8. How many da. are there in 4 wk. 3 da.? In 5 wk.?
9. Which of the following named years are leap years? 1880? 1892? 1900? 2000? 1864? 1870?
10. How many da. are there in June? In Dec.? In Mar.? In Aug.? In Jan.? In Sept.? In Feb.?

COUNTING.

222. The following denominations are used in counting :

TABLE.

12 things	= 1 dozen (doz.).
12 dozen	= 1 gross (gr.).
12 gross	= 1 great gross (G. gr.).
20 things	= 1 score.

STATIONERS' TABLE.

223. The following table is used in the paper trade :

TABLE.

24 sheets	= 1 quire (qr.).
20 quires	= 1 ream (R.).
2 reams	= 1 bundle.
5 bundles	= 1 bale.

EXERCISES.

1. How many things are there in 3 doz. ? In 6 doz. ?
2. How many are 3 score ? 3 score and 10 ?
3. How many doz. are there in 3 gr. ? In 1 G. gr. ?
4. How many sheets are there in 2 qr. ? In $5\frac{1}{2}$ qr. ?
5. How many qr. are there in 3 R. ? In 6 R. ? In $6\frac{1}{4}$ R. ?
6. How many qr. are there in 48 sheets ? In 96 sheets ?

224. MISCELLANEOUS EXERCISES.

- | | | | |
|----------------|---------------|----------------------------|-------------|
| 1. 4 bu. | = ? pk. | 11. $1\frac{1}{2}$ cu. ft. | = ? cu. in. |
| 2. 3 ft. | = ? in. | 12. 32 qt. | = ? pk. |
| 3. 5 lb. | = ? oz. (Ap.) | 13. $2\frac{1}{4}$ A. | = ? sq. rd. |
| 4. 10 qt. | = ? pt. | 14. 3 yd. | = ? in. |
| 5. 18 sq. ft. | = ? sq. yd. | 15. 48 qt. | = ? gal. |
| 6. 6 oz. | = ? pwt. | 16. $3\frac{1}{2}$ hr. | = ? min. |
| 7. 70 da. | = ? wk. | 17. 10 R. | = ? qr. |
| 8. 60 units | = ? doz. | 18. 72 gr. | = ? pwt. |
| 9. 3 qr. | = ? sheets | 19. 32 oz. (Av.) | = ? lb. |
| 10. 54 cu. ft. | = ? cu. yd. | 20. 4 score | = ? units. |

REDUCTION OF DENOMINATE NUMBERS.

225. The process of changing a denominate number from one denomination to another without altering its value is called **Reduction**.

226. The process of changing a denominate number to an equivalent number of a *lower* denomination is called **Reduction to Lower Denominations**, or **Reduction Descending**.

227. The process of changing a denominate number to an equivalent number of a higher denomination is called **Reduction to Higher Denominations**, or **Reduction Ascending**.

REDUCTION TO LOWER DENOMINATIONS.

228. 1. How many inches are there in 7 yd. 1 ft. 4 in. ?

7	
<u>3</u>	
21	EXPLANATION. — Since there are 3 ft. in 1 yd., in 7 yd.
1	there are 7 times 3 ft., or 21 ft., and 21 ft. + 1 ft. = 22 ft.
<u>22</u>	Since there are 12 in. in 1 ft., in 22 ft. there are 22 times 12 in.,
12	or 264 in., and 264 in. + 4 in. = 268 in. Therefore 7 yd. 1 ft. 4
264	in. = 268 in.
4	The business method of reducing denominate numbers to
<u>268</u>	lower denominations is more convenient than that given in the
	explanation. It is illustrated by the process in the margin.

229. Hence the business method is as follows :

To reduce denominate numbers to lower denominations, multiply the number of the highest denomination given by the number indicating how many units of the next lower denomination are equal to one of the higher, and to the product add the number given of this lower denomination.

Proceed in like manner with this and each successive result thus obtained, until the number is reduced to the required denomination.

Reduce to lower denominations :

- | | |
|--------------------------------|-------------------------------------|
| 2. 4 yd. 2 ft. 8 in. | 23. 4 rd. 3 yd. 2 ft. 6 in. |
| 3. 12 yd. 1 ft. 7 in. | 24. 5 rd. 2 yd. 1 ft. 8 in. |
| 4. 15 yd. 2 ft. 10 in. | 25. 8 rd. 1 yd. 2 ft. 10 in. |
| 5. 27 yd. 2 ft. 11 in. | 26. 3 sq. yd. 4 sq. ft. 18 sq. in. |
| 6. 6 gal. 3 qt. 1 pt. | 27. 5 sq. yd. 8 sq. ft. 100 sq. in. |
| 7. 13 gal. 2 qt. 1 pt. | 28. 2 A. 110 sq. rd. 5 sq. yd. |
| 8. 15 gal. 1 qt. 1 pt. 2 gi. | 29. 8 A. 25 sq. rd. 4 sq. yd. |
| 9. 18 gal. 3 qt. 1 pt. 3 gi. | 30. 2 cu. yd. 10 cu. ft. 85 cu. in. |
| 10. 4 hr. 15 min. 15 sec. | 31. 4 cu. yd. 6 cu. ft. 714 cu. in. |
| 11. 10 hr. 12 min. 18 sec. | 32. 3 lb. 2 oz. 7 dr. 1 sc. |
| 12. 18 hr. 20 min. 35 sec. | 33. 4 lb. 8 oz. 5 dr. 2 sc. |
| 13. 4 cwt. 20 lb. 8 oz. | 34. 3 R. 10 qr. 18 sheets. |
| 14. 8 cwt. 75 lb. 12 oz. | 35. 4 R. 8 qr. 10 sheets. |
| 15. 2 T. 5 cwt. 80 lb. 12 oz. | 36. 3 lb. 6 oz. 10 pwt. |
| 16. 4 T. 7 cwt. 35 lb. 9 oz. | 37. 5 lb. 10 oz. 3 sc. |
| 17. 4 lb. 6 oz. 18 pwt. 10 gr. | 38. 6 da. 10 hr. 25 min. |
| 18. 8 lb. 4 oz. 10 pwt. 8 gr. | 39. 8 bu. 3 pk. 7 qt. 1 pt. |
| 19. 5 lb. 5 oz. 15 pwt. 9 gr. | 40. 8 rd. 4 yd. 2 ft. 6 in. |
| 20. 3 bu. 3 pk. 5 qt. 1 pt. | 41. 30 bu. 3 pk. 6 qt. |
| 21. 5 bu. 2 pk. 7 qt. 1 pt. | 42. 5 A. 140 sq. rd. 3 sq. yd. |
| 22. 6 bu. 1 pk. 4 qt. 1 pt. | 43. 7 mi. 100 rd. 13 ft. 6 in. |

44. Reduce .75 yd. to feet and inches.

$$\begin{array}{r}
 .75 \\
 \underline{.3} \\
 2.25 \\
 \underline{.12} \\
 3.00
 \end{array}$$

EXPLANATION. — Since in 1 yd. there are 3 ft., in .75 yd. there are .75 of 3 ft. or 2.25 ft.

Since in 1 ft. there are 12 in., in .25 ft. there are .25 of 12 in., or 3 in.

Therefore, .75 yd. = 2 ft. 3 in.

.75 yd. = 2 ft. 3 in.

- | | | | |
|------------------------|-----------------------|-----------------------|----------------|
| 45. $\frac{3}{8}$ cwt. | 48. $\frac{3}{7}$ yd. | 51. .4 cu. yd. | 54. .73 wk. |
| 46. $\frac{5}{8}$ rd. | 49. .6 bu. | 52. $\frac{5}{8}$ mi. | 55. .3125 gal. |
| 47. $\frac{7}{8}$ wk. | 50. .45 T. | 53. $\frac{3}{7}$ A. | 56. .475 yd. |

REDUCTION TO HIGHER DENOMINATIONS.

230. 1. Reduce 235 pints to bushels, pecks, etc.

$$\begin{array}{r|l}
 2 & 235 \text{ pt.} \\
 8 & 117 \text{ qt.} \quad 1 \text{ pt.} \\
 4 & 14 \text{ pk.} \quad 5 \text{ qt.} \\
 & 3 \text{ bu.} \quad 2 \text{ pk.}
 \end{array}$$

EXPLANATION. — Since there are 2 pt. in 1 qt., in 235 pt. there are as many quarts as 2 pt. are contained times in 235 qt., or 117 qt. 1 pt.

235 pt. = 3 bu. 2 pk. 5 qt. 1 pt.

Since there are 8 qt. in 1 pk., there are as many pecks in 117 qt.

as 8 qt. are contained times in 117 qt., or 14 pk. 5 qt.

Since there are 4 pk. in 1 bu., in 14 pk. there are as many bushels as 4 pk. are contained times in 14 pk., or 3 bu. 2 pk.

Therefore, 235 pt. are equal to 3 bu. 2 pk. 5 qt. 1 pt.

231. Hence it is evident that:

To reduce a denominate number to units of higher denominations, the given number is divided by the number indicating how many units of the given denomination make one of the next higher denomination.

We proceed in like manner with this, and each successive quotient, till the whole is reduced to the required denomination.

The last quotient, with the remainders (when there are any) annexed, will be the required answer.

2. Reduce 3265 inches to rods, yards, etc.

$$\begin{array}{r|l}
 12 & 3265 \text{ in.} \\
 3 & 272 \text{ ft.} \quad 1 \text{ in.} \\
 5\frac{1}{2} & 90 \text{ yd.} \quad 2 \text{ ft.} \\
 & 2 \\
 11 & 180 \quad [\text{or } 2 \text{ yd.}] \\
 & 16 \text{ rd.} \quad 4 \text{ half-yd.,}
 \end{array}$$

SUGGESTION. — In dividing by $5\frac{1}{2}$ it is usually more convenient to reduce the divisor to halves and the dividend also to halves before dividing.

It should be remembered then, that the remainder, if there is any, is *half-yards*, which must be changed to yards.

Reduce:

3. 4213 gi. to gal., etc.

6. 3539 in. to rd., etc.

4. 3185 pt. to gal., etc.

7. 2456 in. to rd., etc.

5. 5724 gi. to gal., etc.

8. 1605 in. to rd., etc.

- | | |
|----------------------------|-----------------------------------|
| 9. 3564 pt. to bu., etc. | 18. 9894 min. to da., etc. |
| 10. 4683 pt. to bu., etc. | 19. 3943 sq. in. to sq. yd., etc. |
| 11. 3947 pt. to bu., etc. | 20. 5937 sq. in. to sq. yd., etc. |
| 12. 5164 oz. to cwt., etc. | 21. 1894 sheets to R., etc. |
| 13. 8127 oz. to cwt., etc. | 22. 2315 sheets to R., etc. |
| 14. 5983 lb. to tons, etc. | 23. 7864 cu. in. to cu. ft., etc. |
| 15. 3795 lb. to tons, etc. | 24. 6754 gr. to lb. Troy, etc. |
| 16. 4285 hr. to wk., etc. | 25. 8386 gr. to lb. Troy, etc. |
| 17. 8168 sec. to hr., etc. | 26. 9138 gr. to lb. Apoth., etc. |

27. Reduce 2 qt. 1 pt. to the fraction of a gallon.

EXPLANATION. — 2 qt. 1 pt. = 5 pt. ; 1 gal. = 8 pt. Therefore 2 qt. 1 pt., or 5 pt., are $\frac{5}{8}$ of 1 gal., or 8 pt.

28. Reduce 1 ft. 3 in. to the fraction of a yard.
 29. Reduce 3 qt. 1 pt. to the fraction of a peck.
 30. Reduce 4 pk. 3 qt. to the fraction of a bushel.
 31. Reduce 2 qt. 1 pt. 3 gi. to the fraction of a gallon.
 32. Reduce 2 pk. 4 qt. 1 pt. to the fraction of a bushel.
 33. Reduce 4 yd. 2 ft. 6 in. to the fraction of a rod.

ADDITION OF DENOMINATE NUMBERS.

232. 1. Find the sum of 3 gal. 3 qt. 1 pt. ; 5 gal. 2 qt. 1 pt. ; 6 gal. 2 qt. 1 pt. ; 8 gal. 3 qt.

gal.	qt.	pt.	
3	3	1	EXPLANATION. — The numbers are written so that units of the same denomination stand in the same column, and we begin at the lowest denomination to add.
5	2	1	The sum of the pints is 3 pt., which are equal to 1 qt.
6	2	1	1 pt. ; the 1 pt. is written under pints and the 1 qt. reserved to add to the quarts.
8	3		
24	3	1	The sum of the quarts is 11 qt., which are equal to 2 gal. 3 qt. ; the 3 qt. are written under the quarts and the 2 gal. reserved to add to the gallons.

The sum of the gallons is 24 gal., which are written under the gallons.

Find the sum of:

2. 3 gal. 3 qt. 1 pt.; 18 gal. 2 qt. 1 pt.; 12 gal. 3 qt.
3. 8 bu. 3 pk. 5 qt.; 16 bu. 2 pk. 7 qt.; 12 bu. 3 pk. 4 qt.
4. 7 da. 5 hr. 20 min.; 8 da. 14 hr. 10 min.; 6 da. 18 hr.
5. 24 lb. 6 oz. (Av.); 35 lb. 14 oz.; 26 lb. 10 oz.; 83 lb.
6. 13 lb. 4 oz. 10 pwt. 8 gr.; 11 lb. 5 oz. 11 pwt. 16 gr.;
10 lb. 10 oz. 15 pwt. 21 gr.
7. 10 yd. 2 ft. 10 in.; 15 yd. 1 ft. 9 in.; 8 yd. 2 ft. 7 in.;
18 yd. 1 ft. 11 in.; 16 yd. 2 ft. 8 in.
8. 15 gal. 3 qt. 1 pt. 3 gi.; 18 gal. 2 qt. 1 pt. 2 gi.; 28
gal. 2 qt.; 21 gal. 1 pt. 3 gi.
9. 12 A. 35 sq. rd.; 14 A. 110 sq. rd.; 15 A. 132 sq. rd.;
11 A. 96 sq. rd.; 25 A. 100 sq. rd.
10. 5 T. 6 cwt. 14 lb. 10 oz.; 7 T. 15 cwt. 36 lb. 15 oz.;
17 T. 8 cwt. 84 lb. 12 oz.; 10 T. 9 cwt. 94 lb. 11 oz.
11. 5 yd. 2 ft. 8 in.; 4 yd. 1 ft. 11 in.; 4 yd. 2 ft. 9 in.;
3 yd. 1 ft.; 4 yd. 8 in.; $\frac{3}{8}$ of a yard and $\frac{1}{4}$ of a foot.

SUBTRACTION OF DENOMINATE NUMBERS.

233. 1. From 12 lb. 6 oz. 7 pwt. take 3 lb. 8 oz. 10 pwt.

lb.	oz.	pwt.	EXPLANATION.
12	6	7	— The numbers are written so that
3	8	10	units of the same denomination stand in the same
8	9	17	column, and we begin at the lowest denomination to
			subtract. Since 10 pwt. cannot be subtracted from
			7 pwt., a unit of the next higher order is united with
			the 7 pwt., making 1 oz. 7 pwt., or 27 pwt. Then 27 pwt. — 10 pwt. =
			17 pwt., which we write under pwt. in the remainder.

Since 1 oz. was united with the 7 pwt., only 5 oz. are left in the minuend. Inasmuch as 8 oz. cannot be subtracted from 5 oz., a unit of the next higher order is united with it, making 1 lb. 5 oz., or 17 oz. Then 17 oz. — 8 oz. = 9 oz., which we write under oz. in the remainder.

Since 1 lb. was changed to ounces, there are but 11 lb. left in the minuend. Then 11 lb. — 3 lb. = 8 lb.

Therefore, the remainder is 8 lb. 9 oz. 17 pwt.

2. From 8 bu. 4 pk. 3 qt. take 3 bu. 2 pk. 7 qt.

3. From 12 gal. 2 qt. 1 pt. 2 gi. take 5 gal. 3 qt. 1 pt. 3 gi.
4. From 10 da. 5 hr. 20 min. take 7 da. 7 hr. 10 min.
5. From 31 lb. 4 oz. 15 pwt. take 8 lb. 6 oz. 12 pwt.
6. From 15 yd. 2 ft. 7 in. take 4 yd. 2 ft. 10 in.
7. From 12 rd. 4 yd. 1 ft. take 8 rd. 4 yd. 2 ft.
8. From 13 lb. 8 oz. 5 dr. take 7 lb. 10 oz. 7 dr.
9. From 18 A. 18 sq. rd. take 7 A. 110 sq. rd.
10. From 40 sq. yd. 8 sq. ft. 51 sq. in. take 15 sq. yd. 6 sq. ft. 100 sq. in.
11. From 25 T. 8 cwt. 75 lb. 10 oz. take 10 T. 11 cwt. 35 lb. 15 oz.
12. How many years, months, and days intervened between Dec. 24, 1892, and May 26, 1842 ?

yr.	mo.	da.	EXPLANATION.
1892	12	24	Since the later date expresses the greater period of time, it is written as the minuend, and the earlier date is, therefore, the subtrahend. In writing the dates the number of the month is given instead of its name. The subtraction is then performed as in denominate numbers, considering a month as 30 days, and a year as 12 months.
1842	5	26	
50	6	28	

13. How long was it from Dec. 10, 1835, to May 15, 1887 ?
14. How long was it from June 13, 1864, to Jan. 1, 1890 ?
15. How long was it from Apr. 8, 1839, to July 7, 1891 ?
16. How long was it from Sept. 23, 1881, to Aug. 10, 1892 ?

MULTIPLICATION OF DENOMINATE NUMBERS.

234. 1. Multiply 3 bu. 3 pk. 7 qt. by 7.

bu.	pk.	qt.	EXPLANATION.
3	3	7	7 times 7 qt. are 49 qt., or 6 pk. 1 qt. The 1 qt. is written under qt. in the product, and the 6 pk. reserved to add to the pecks. 7 times 3 pk. are 21 pk.; 21 pk. + 6 pk. reserved are 27 pk., or 6 bu. 3 pk. The 3 pk. are written under pk. in the product, and the 6 bu. reserved to add to the bushels. Seven times 3 bu. are 21 bu.; 21 bu. + 6 bu. reserved are 27 bu. Therefore, the product is 27 bu. 3 pk. 1 qt.
27	3	1	

2. Multiply 4 bu. 3 pk. 5 qt. by 7.
3. Multiply 5 gal. 1 qt. 3 pt. 2 gi. by 6.
4. Multiply 5 lb. 6 oz. 10 pwt. 8 gr. by 8.
5. Multiply 7 lb. 8 oz. 5 dr. 2 sc. 10 gr. by 7.
6. Multiply 3 hr. 20 min. 35 sec. by 5.
7. Multiply 2 T. 5 cwt. 48 lb. 15 oz. by 8.
8. Multiply 2 rd. 3 yd. 2 ft. 10 in. by 9.
9. Multiply 12 cu. yd. 15 cu. ft. 1115 cu. in. by 6.
10. Multiply 8 sq. yd. 2 sq. ft. 45 sq. in. by 5.
11. Multiply 5 da. 8 hr. 15 min. 25 sec. by 4.
12. Multiply 12 rd. 4 yd. 2 ft. 8 in. by 8.
13. Multiply 5 R. 8 qr. 16 sheets by 9.
14. Multiply 6 bar. 10 gal. 3 qt. 1 pt. by 7.

DIVISION OF DENOMINATE NUMBERS.

- 235.** 1. Divide 14 gal. 3 qt. 1 pt. by 4.

gal.	qt.	pt.	EXPLANATION.—
4)14	3	1	One fourth of 14 gal. is 3 gal. and 2 gal. remainder. The 3 gal. are written in the quotient, and the 2 gal. remainder are united with the 3 qt., making 11 qt.
	3	2 1 $\frac{1}{2}$	

One fourth of 11 qt. is 2 qt. and 3 qt. remainder. The 2 qt. are written in the quotient, and the 3 qt. remainder united with the 1 pt., making 7 pt.

One fourth of 7 pt. is 1 $\frac{1}{2}$ pt.

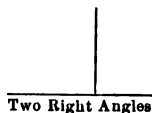
Therefore, the quotient is 3 gal. 2 qt. 1 $\frac{1}{2}$ pt.

2. Divide 21 gal. 3 qt. 1 pt. 3 gi. by 6.
3. Divide 15 bu. 3 pk. 5 qt. 1 pt. by 4.
4. Divide 13 yd. 2 ft. 6 in. by 7.
5. Divide 23 cwt. 68 lb. 10 oz. by 5.
6. Divide 42 lb. 8 oz. 15 pwt. by 9.
7. Divide 22 lb. 7 oz. 5 dr. 2 sc. by 8.
8. Divide 19 hr. 10 min. 36 sec. by 6.
9. Divide 21 sq. yd. 7 sq. ft. 45 sq. in. by 4.
10. Divide 15 rd. 4 yd. 2 ft. 8 in. by 5.

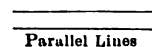
PRACTICAL MEASUREMENTS.

236. When a straight line meets another straight line forming two equal angles, each angle is called a **Right Angle**.

When two lines form right angles, they are said to be *perpendicular* to each other.



237. Lines equidistant throughout their entire length are called **Parallel Lines**.

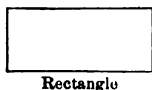


238. A figure having four straight sides and its opposite sides parallel is called a **Parallelogram**.



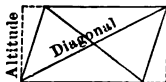
1. When the angles of a parallelogram are right angles, it is called a **Rectangle**.

2. The side upon which a figure is assumed to stand is called the **Base**.



3. The perpendicular distance between the base of a figure and the highest point opposite it is the **Altitude**.

4. The straight line joining the opposite angles of a parallelogram is called its **Diagonal**.



239. To compute the area of a rectangular surface.

1. What is a square inch? Draw a figure 6 in. long and 1 in. wide with all its angles equal. How many sq. in. does it contain?

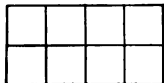
2. If the figure were 2 in. wide, how many sq. in. would it contain? How many, if it were 3 in. wide? 4 in. wide? 5 in. wide? 6 in. wide?

3. What is a square foot? How many sq. ft. are there in a strip of ground 12 ft. long and 1 ft. wide, with all its angles equal? How many, if it is 2 ft. wide? 3 ft. wide? 4 ft. wide? 5 ft. wide? 6 ft. wide?

4. How many sq. yd. are there in a walk 15 yd. long and 5 yd. wide, having its angles all equal?

240. The number of square units that a surface contains is called its **Area**.

Thus, the area of a rectangle 4 in. long and 2 in. wide is 8 sq. in., for it may be divided into 2 rows, each containing 4 sq. in.



WRITTEN EXERCISES.

241. 1. What is the area of a rectangular walk 22 ft. long and 5 ft. 6 in. broad?

$$22 \times 5\frac{1}{2} = 121$$

Or,

$$264 \times 66 = 17,424$$

$$17,424 \text{ sq. in.} = 121 \text{ sq. ft.}$$

EXPLANATION. — Since the walk is 22 ft. long and $5\frac{1}{2}$ ft. wide, its area in sq. ft. is $22 \times 5\frac{1}{2}$, or 121 sq. ft.

The same result can be obtained by expressing the length and breadth in inches, and reducing the result obtained to square feet.

242. Hence it is evident that:

The area of any rectangular surface is obtained by finding the product of the numbers expressing its length and breadth.

The length and breadth must be expressed in units of the *same denomination*.

2. What is the area of the floor of a rectangular room 17 ft. long and 23 ft. wide?

3. How many sq. ft. of surface are there in the side wall of a room 16 ft. long and 12 ft. high? How many sq. yd.?

4. The floor of a rectangular room 18 ft. wide and 24 ft. long contains how many sq. yd.?

5. A farm in the form of a rectangle has a length of 160 rd. and a breadth of 120 rd. How many sq. rd. of land does it contain? How many acres?

6. A farmer paid \$2.50 per acre for plowing a rectangular field that was 25 rd. wide and 45 rd. long. What was the expense of plowing the field?

7. If half of a rectangular field, 80 rd. long and 60 rd. wide, is planted with corn and half of the remainder with potatoes, how many acres are planted with each?

243. To compute the area of a parallelogram.

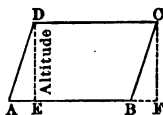
1. How does the figure *AED* compare in area with the figure *BFC*?

2. Since the figure *EBCD* is common to the parallelogram *ABCD* and the rectangle *EFCD*, how does *ABCD* compare in area with *EFCD*?

3. How does the base *AB* compare in length with the base *EF*?

4. How does the altitude *DE* of the parallelogram compare with the breadth, or altitude, of the rectangle?

5. How does a parallelogram compare in area with a rectangle having the same base and altitude?


WRITTEN EXERCISES.

244. 1. What is the area of a parallelogram whose base is 24 ft. and altitude 7 ft. 3 in.?

$$24 \times 7\frac{3}{4} = 174$$

Or,

$$288 \times 87 = 25,056$$

$$25,056 \text{ sq. in.} = 174 \text{ sq. ft.}$$

EXPLANATION. — Since a parallelogram is equal in area to a rectangle having the same base and altitude, and since the base is 24 ft. and the altitude $7\frac{3}{4}$ ft., § 242, the area in sq. ft. is $24 \times 7\frac{3}{4}$, or 174 sq. ft.

The same result may be obtained by expressing both the base and the altitude in inches, and reducing the result to square feet.

245. Hence it is evident that:

The area of a parallelogram is equal to the product of the numbers expressing its base and altitude.

The base and altitude must be expressed in units of the *same denomination*.

2. What is the area of a parallelogram whose base is 20 ft. and altitude 6 ft.?

3. What is the area of a parallelogram whose base is 125 rd. and altitude 60 rd.?

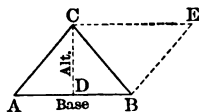
4. What is the area of a parallelogram whose base is 25 ft. 4 in. and altitude 12 ft.?

5. How many acres are there in a farm in the form of a parallelogram whose base is 128 rd. and altitude 120 rd.?

6. The area of a parallelogram is 128 sq. ft. and its base is 16 ft. What is its altitude?

246. To compute the area of a triangle.

247. A figure having three straight sides and three angles is called a **Triangle**.



1. How does the triangle ABC compare with the triangle BEC ? Then, what part of the parallelogram $ABEC$ is the triangle ABC ?

2. How does the base of the parallelogram compare with the base of the triangle? The altitude of the parallelogram with the altitude of the triangle?

3. How does the area of a triangle compare with the area of a parallelogram having the same base and altitude?

WRITTEN EXERCISES.

248. 1. What is the area of a triangle whose base is 20 ft. and altitude 16 ft.?

$\frac{1}{2}$ of $20 \times 16 = 160$
Area of triangle = 160 sq. ft.

EXPLANATION. — Since the area of a triangle is equal to one half the area of a parallelogram having the same base and altitude,

and since the base is 20 ft. and the altitude 16 ft., § 245, the area in square feet is $\frac{1}{2}$ of 20×16 , or 160 sq. ft.

249. Hence it is evident that:

The area of a triangle is one half the product of the numbers expressing its base and altitude.

The base and altitude must be expressed in units of the *same denomination*.

2. What is the area of a triangle whose base is 10 in. and altitude 6 in.?

3. What is the area of a triangle whose base is 15 yd. and altitude 8 yd.?

4. What is the area of a triangle whose base is 19 ft. and altitude 28 ft. ?

5. What is the area of a triangle whose base is 54 rd. and altitude 40 rd. ?

6. How many acres are there in a triangular field whose base is 80 rd. and altitude 28 rd. ?

7. The area of a triangle is 450 sq. ft. and its base is 36 ft. What is the altitude of the triangle ?

250. To find the cost of plastering, painting, etc.

Plastering, painting, kalsomining, etc., are usually computed by the square yard.

Custom varies greatly as to the allowances that should be made for openings in walls, such as doors and windows. Sometimes their whole area is deducted, sometimes half their area is deducted, and sometimes no allowance is made ; consequently, to avoid complications at the time of settlement, a written contract regarding allowances should be made.

WRITTEN EXERCISES.

251. 1. A plasterer received 23 cents per square yard for plastering the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 12 ft. high, no deduction being made for openings. How much did he receive ?

2. Find the entire cost of kalsomining the ceiling of a hall 26 ft. long and 6 ft. wide, and of a room 18 ft. long and 16 ft. wide, at 9 cents per square yard.

3. A rectangular athletic field 154 yd. by 165 yd. has a close board fence around it $7\frac{1}{2}$ ft. high. Find the cost of painting the outside of the fence at 6 cents per square yard.

4. Find the cost of plastering the walls and ceiling of a room 16 ft. long, 12 ft. wide, and 9 ft. high, at 27 cents per square yard, allowing for the base board, which is 9 in. wide, and for one half the area of the openings, there being one door 3 ft. by 7 ft., and 2 windows, each 3 ft. by 6 ft.

252. To find the cost of carpeting.

Carpets are sold by the lineal yard, and they are usually either 1 yard or $\frac{3}{4}$ of a yard in width; but matting, oilcloth, and other materials vary in width.

1. In matching the patterns in carpets there is usually some waste.
2. Sometimes it is necessary to make a carpet a little too wide and *turn it under*, because the width of a strip may not be exactly contained in the width of the room, and parts of strips cannot be purchased; consequently, in computing the cost of carpets, the number of strips necessary to cover the floor must be found.

WRITTEN EXERCISES.

253. 1. How many yards of carpet, one yard wide, will be required for a room 22 ft. long and 17 ft. wide, if the strips run lengthwise and there is no waste in matching?

SOLUTION. — Since the width of the room is 17 ft., 6 strips will be required, although a portion must be turned under or cut off. Each strip being $7\frac{1}{2}$ yards long, 44 yards will be required.

2. A room 18 ft. wide and 24 ft. long was carpeted with carpet 1 yd. wide without waste in matching. How much did the carpet cost at \$1.25 per lineal yard?

3. How much will it cost to carpet a room 27 ft. long and 18 ft. wide with carpet $\frac{3}{4}$ of a yard wide at \$.80 per lineal yard, if there is no waste in matching?

4. How many yards of carpet, one yard wide, will be required for a room 30 ft. long and $26\frac{1}{2}$ ft. wide, if the strips run lengthwise and there is no waste in matching?

5. What will be the entire cost of carpeting the room, if the carpet costs \$1.05 a lineal yard and 5 cents a yard for putting it down, and the floor is first covered with paper lining at 9 cents a square yard?

6. How many yards of carpet, 27 in. wide, will be required for a room 18 ft. long and 15 ft. wide, if the strips run lengthwise and there is a waste of $\frac{1}{7}$ of a yard in each strip in matching the pattern?

254. To find the cost of papering.

Wall paper is sold by the roll, and in computations any part of a roll is considered a whole roll.

A roll is 8 yards long and 18 inches wide, unless otherwise specified.

1. Paper is often put up in *double* rolls, 16 yards long, to avoid waste in cutting. Double rolls are counted as two rolls each.

2. Borders or friezes are sold by the yard, and are of various widths.

WRITTEN EXERCISES.

255. 1. Find the number of rolls of paper required to paper the walls of a room 15 ft. long, 12 ft. wide, and having a height above the baseboard of $7\frac{3}{4}$ ft., allowing for one door 3 ft. by $6\frac{1}{2}$ ft. and two windows 3 ft. by 5 ft.

SOLUTION. — The distance around the room is 18 yd., therefore, 2 times 18, or 36, strips will be required, if there are no deductions for openings; but the door and windows will make a difference of *about* 4 strips, leaving 32 to be cut. Since a single roll is 8 yd. long, 3 strips can be cut from a single roll. Hence, it will take $10\frac{2}{3}$ rolls; but since a part of a roll is considered a whole roll, 11 rolls at least will be needed.

NOTE. — Inasmuch as there is waste in matching patterns, also in fractional parts of strips and rolls, it is rarely possible to find the exact cost of papering, but the method given above will closely approximate accuracy.

2. How many rolls of paper will be required to paper the walls of a room 16 ft. by 12 ft. and 8 ft. high, making no deductions?

3. How many rolls of paper will be required to paper the ceiling of the same room, if the strips run crosswise?

4. How many rolls of paper, put up in double rolls, will be required to paper the walls of a room 24 ft. by 18 ft. and $9\frac{1}{2}$ ft. high above the baseboard, deducting for one door 3 ft. by 7 ft. and two windows each 3 ft. by 6 ft.?

5. How many rolls will be required for the ceiling of this room, strips running lengthwise, and what will it cost to paper the room at \$.40 per single roll?

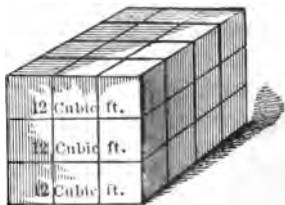
256. To compute the volume of rectangular solids.

1. What is a cubic inch? A cubic foot? A cubic yard?
2. How many cubic feet of stone are there in a rectangular block of stone which is 6 ft. long, 1 ft. wide, and 1 ft. thick?
3. How many cubic feet would there be if the block were 2 ft. wide? 2 ft. thick? 2 ft. wide and 2 ft. thick? How many if it were 7 ft. long, 2 ft. wide, and 2 ft. thick?
4. How many cubic feet are there in a similar block which is 6 ft. long, 3 ft. wide, and 1 ft. thick? In a block 6 ft. long, 3 ft. wide, and 2 ft. thick? In a block 6 ft. long, 3 ft. wide, and 3 ft. thick? In a block 6 ft. long, 4 ft. wide, and 3 ft. thick? In a block 6 ft. long, 3 ft. wide, and 4 ft. thick? In a block 3 ft. long, 4 ft. wide, and 6 ft. thick?

257. The number of solid units that a body contains is its **Volume** or **Solid contents**.

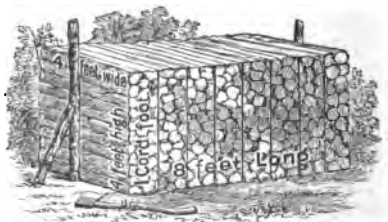
The solid units are cubic inches, cubic feet, cubic yards, etc.

If a solid is 4 ft. long, 3 ft. wide, and 3 ft. thick, its volume is 36 cu. ft., for it may be divided into 3 blocks, each containing 12 cu. ft., as shown by the illustration.



258. A pile of wood or stone 8 ft. long, 4 ft. wide, and 4 ft. high is called a **Cord**.

A pile that is 1 ft. long, 4 ft. wide, and 4 feet high is called a **Cord Foot**.



Stone walls are sometimes measured by the **Perch**, which contains 24.75 cu. ft.

WRITTEN EXERCISES.

259. 1. How many cubic feet of stone are there in a rectangular block 8 ft. long, 5 ft. 3 in. wide, and 3 ft. 6 in. thick?

$8 \times 5\frac{1}{4} \times 3\frac{1}{2} = 147$ **EXPLANATION.** — Since a block 8 ft. long, 1 ft. wide, and 1 ft. thick contains 8 cu. ft., a block 5 $\frac{1}{4}$ ft. wide and 1 ft. thick contains 5 $\frac{1}{4}$ times 8 cu. ft., or 42 cu. ft.

Since a block 8 ft. long, 5 $\frac{1}{4}$ ft. wide, and 1 ft. thick, contains 42 cu. ft., a similar block, that is 3 $\frac{1}{2}$ ft. thick, contains 3 $\frac{1}{2}$ times 42 cu. ft., or 147 cu. ft.

260. Hence it is evident that:

The volume or solid contents of a rectangular solid are obtained by finding the product of the numbers expressing its length, breadth, and thickness.

The length, breadth, and thickness must be expressed in units of the same denomination.

Find the volume of solids with dimensions as follows:

2. Length 10 ft., breadth 7 ft., thickness 6 ft.
3. Length 9 in., breadth 11 in., thickness 8 in.
4. Length 5 yd., breadth 8 yd., thickness 10 yd.
5. Length 14 ft., breadth 10 ft., thickness 15 ft.
6. Length 8 ft. 6 in., breadth 8 ft., thickness 10 ft.
7. Length 12 ft. 4 in., breadth 9 ft., thickness 8 ft.
8. Length 10 ft. 6 in., breadth 10 ft., thickness 12 ft.
9. Length 12 ft. 4 in., breadth 10 ft., thickness 8 ft.
10. Length 8 ft. 4 in., breadth 10 ft. 6 in., thickness 8 ft.

How many cords of wood are there in the following piles?

11. Length 24 ft., breadth 4 ft., height 4 ft.
12. Length 32 ft., breadth 4 ft., height 8 ft.
13. Length 48 ft., breadth 16 ft., height 12 ft.
14. How much will the wood in the last pile cost at \$5.50 per cord?

15. How much will it cost to excavate a cellar which is 40 ft. long, 22 ft. wide, and 8 ft. deep, at 30¢ per cubic yard?

16. How much will be the expense of laying a stone wall 40 ft. long, 1 ft. 6 in. thick, and 6 ft. high, at \$2.50 per perch?

261. To measure lumber.

In measuring lumber, boards 1 inch thick or less are estimated by the number of square feet of surface.

Thus a board 1 foot wide and 10 feet long contains 10 feet *board measure*, if it is 1 inch or less in thickness.

The length and width must both be expressed in feet.

262. When lumber is more than 1 inch in thickness, the number of feet board measure is obtained as follows:

The length in feet is multiplied by the breadth in feet, and this product by the number of inches in thickness.

Thus, the number of feet *board measure* in a timber 12 ft. long, 15 in. wide, and $2\frac{1}{2}$ in. thick is obtained as follows:

$$12 \times 1\frac{1}{2} \times 2\frac{1}{2} = 37\frac{1}{2}, \text{ the number of feet board measure.}$$

How many feet are there in the following boards:

1. 16 ft. long, 18 in. wide.
4. 20 ft. long, 14 in. wide.
2. 12 ft. long, 15 in. wide.
5. 15 ft. long, 10 in. wide.
3. 18 ft. long, 9 in. wide.
6. 20 ft. long, 16 in. wide.
7. What will be the cost of 10 boards, each 12 ft. long and 15 in. wide, at \$20 per thousand feet?
8. How many feet board measure are there in 20 joists 18 ft. long, 16 in. wide, and 3 in. thick?
9. What will those joists cost at \$18 per thousand feet?
10. What will be the cost of 20 planks 16 ft. long, 14 in. wide, and 2 in. thick, at \$22 per thousand feet?
11. What will be the cost of 20 planks 12 ft. long, 15 in. wide, and $1\frac{1}{2}$ in. thick, and 40 scantlings 6 in. wide, 12 ft. long and 4 in. thick, at \$18 per thousand feet?

263. To find the capacity of rectangular bins, boxes, etc.**WRITTEN EXERCISES.**

1. How many bushels of grain will a bin hold that is 5 ft. long, 4 ft. wide, and 6 ft. high?

$$4 \times 5 \times 6 = 120$$

$$1728 \times 120 = 207,360$$

$$207,360 \div 2150.4 = 96.428 +$$

2150.4 cu. in., the bin will hold as many bushels as 2150.4 is contained times in 207,360, or 96.428+ bu.

EXPLANATION. — Since the bin is 5 ft. long, 4 ft. wide, and 6 ft. high, § 260, its volume is $5 \times 4 \times 6$, or 120 cu. ft., or 207,360 cu. in.

Since, § 217, 1 bu. contains

Find the contents in bushels of the following:

2. A box 4 ft. long, 3 ft. wide, and $3\frac{1}{2}$ ft. high.
3. A bin 6 ft. long, 5 ft. wide, and $5\frac{1}{2}$ ft. high.
4. A bin 10 ft. long, 8 ft. wide, and $5\frac{1}{2}$ ft. high.
5. How many bushels of grain are there in a bin that is 5 ft. square, and 6 ft. high, if it is half full?

264. To find the capacity of rectangular tanks, cisterns, etc.**WRITTEN EXERCISES.**

1. How many gallons will a tank hold that is 5 ft. long, 4 ft. wide, and 3 ft. deep?

$$5 \times 4 \times 3 = 60$$

$$1728 \times 60 = 103,680$$

$$103,680 \div 231 = 448.831 +$$

EXPLANATION. — Since the tank is 5 ft. long, 4 ft. wide, and 3 ft. deep, § 260, its volume is $5 \times 4 \times 3$, or 60, cu. ft., or 103,680 cu. in.

Since, § 216, 1 gal. contains 231 cu. in., the tank will hold as many gallons as 231 is contained times in 103,680, or 448.831+ gal.

Find the capacity in gallons of the following:

2. A tank 5 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. deep.
3. A cistern 6 ft. long, 5 ft. wide, and $4\frac{1}{2}$ ft. deep.
4. A cistern $5\frac{1}{2}$ ft. long, 5 ft. wide, and 4 ft. deep.
5. How many barrels of water are there in a cistern 8 ft. long, 6 ft. wide, and 5 ft. deep, if it is one third full?

PERCENTAGE.

265. 1. From a school of 100 pupils 5 were absent. What part of the whole number of pupils was absent?

2. A man's wages were \$ 20 a week and he paid \$ 5 a week for his board. What part of his wages did he pay for board?

How many hundredths of his wages did he pay for board?

3. A regiment of soldiers engaged in battle lost 1 out of every 10 men. How many was that *per hundred*, or *per cent*?

4. In a pasture 7 out of every 10 animals are sheep. How many hundredths of the whole number of animals are the sheep? What per cent of the whole number of animals are the sheep?

5. What is $\frac{4}{100}$, or 4 per cent, of \$ 200? Of \$ 800? Of \$ 4000? Of \$ 7000?

6. What is .06, or 6 per cent, of \$ 500? Of \$ 900? Of \$ 1200? Of \$ 2000?

7. What fractional part of anything is 50 per cent of it? 25 per cent of it? 75 per cent of it? 40 per cent of it?

8. How many hundredths of anything is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{10}$ of it? Then, what per cent of anything is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{10}$ of it?

266. The expression **Per Cent** means *by the hundred*.

Thus, 5 *per cent* means 5 out of every hundred, or 5 hundredths.

267. The **Sign of Per Cent** is %.

Thus, 6% is read 6 per cent.

268. Since per cent is a number of hundredths, it is usually expressed as a decimal. It may also be expressed by a common fraction.

Thus, 4 per cent is written 4%, .04, or $\frac{4}{100}$.

10 per cent is written 10%, .10, or $\frac{10}{100}$.

12½ per cent is written 12½%, .12½, .125, $\frac{12\frac{1}{2}}{100}$, or $\frac{25}{200}$.

$\frac{3}{4}$ per cent is written $\frac{3}{4}$ %, .00¾, .0075, $\frac{\frac{3}{4}}{100}$, or $\frac{3}{400}$.

125 per cent is written 125%, 1.25, or $1\frac{25}{100}$.

Express decimally:

- | | | |
|---------|----------|----------------------|
| 1. 10%. | 5. 6½%. | 9. 135%. |
| 2. 25%. | 6. 8¼%. | 10. 237½%. |
| 3. 44%. | 7. 16¾%. | 11. ¼%. |
| 4. 75%. | 8. 115%. | 12. $\frac{5}{8}$ %. |

Express by common fractions in their lowest terms:

- | | | |
|---------|----------|----------------------|
| 1. 4%. | 5. 8⅓%. | 9. 125%. |
| 2. 5%. | 6. 16⅔%. | 10. 250%. |
| 3. 20%. | 7. 37½%. | 11. ¼%. |
| 4. 25%. | 8. 87½%. | 12. $\frac{5}{8}$ %. |

Express in per cent with the sign:

- | | | | |
|---------------------|---------------------|----------------------|------------------------|
| 1. $\frac{1}{2}$. | 5. $\frac{1}{20}$. | 9. $\frac{7}{10}$. | 13. $\frac{1}{8}$. |
| 2. $\frac{1}{4}$. | 6. $\frac{1}{25}$. | 10. $\frac{8}{25}$. | 14. $\frac{5}{16}$. |
| 3. $\frac{1}{5}$. | 7. $\frac{2}{5}$. | 11. $\frac{1}{3}$. | 15. $\frac{3}{40}$. |
| 4. $\frac{1}{10}$. | 8. $\frac{3}{4}$. | 12. $\frac{1}{6}$. | 16. $\frac{16}{125}$. |

Problems in percentage involve the following elements:

269. The number of which the per cent is to be found is called the **Base**.

270. The number of hundredths taken is called the **Rate**.

271. The number which is a certain number of hundredths of the base is called the **Percentage**.

The base is represented by B, the rate by R, and the percentage by P.

272. To find the percentage when the base and rate are given.

1. Find 5%, or .05, of \$ 100.
2. Find 25%, or .25, of \$ 400.
3. Find 50%, or $\frac{50}{100}$, or $\frac{1}{2}$, of \$ 800.
4. Find $33\frac{1}{3}\%$, or $\frac{33\frac{1}{3}}{100}$, or $\frac{1}{3}$, of \$ 150.

Find:

- | | |
|--------------------|------------------------------------|
| 5. 10% of 200 bu. | 10. 20% of 300 cows. |
| 6. 25% of 300 gal. | 11. $12\frac{1}{2}\%$ of 240 hens. |
| 7. 50% of 800 yd. | 12. 15% of 400 sheep. |
| 8. 70% of 500 mi. | 13. $8\frac{1}{3}\%$ of 144 books. |
| 9. 30% of 400 lb. | 14. 80% of 120 slates. |
15. Henry had 80 cents and spent 30% of his money for a ball. How much did the ball cost?
16. John had 200 marbles and lost 25% of them. How many marbles did he lose?
17. A man who had \$ 400 paid 20% of it for a horse. How much money did he have left?

WRITTEN EXERCISES.

273. 1. What is 25% of \$ 536.48?

$$\text{\$ } 536.48 \times .25 = \text{\$ } 134.12,$$

Or,

$$\frac{1}{4} \text{ of } \text{\$ } 536.48 = \text{\$ } 134.12.$$

EXPLANATION. — Since 25% of a number is .25 of it, 25% of \$ 536.48 is .25 of \$ 536.48, or \$ 134.12.

Or, since 25% of a number is $\frac{25}{100}$, or $\frac{1}{4}$, of it, 25% of \$ 536.48 is $\frac{1}{4}$ of \$ 536.48, or \$ 134.12.

274. Hence it is evident that $B \times R = P$.

Find:

- | | |
|-----------------------|------------------------------------|
| 2. 6% of \$ 140.50. | 6. $33\frac{1}{3}\%$ of \$ 644.64. |
| 3. 80% of \$ 915.20. | 7. $87\frac{1}{2}\%$ of \$ 905.76. |
| 4. 46% of \$ 724.50. | 8. $\frac{1}{2}\%$ of \$ 488.20. |
| 5. 130% of \$ 691.45. | 9. $1\frac{1}{8}\%$ of \$ 500.08. |

10. There were 455 sheep in a pasture, but 20% of them jumped out. How many sheep jumped out?

11. A man who had a pile of wood containing 234 cords sold $16\frac{2}{3}\%$ of it. How many cords did he have left?

12. What is the value of $66\frac{2}{3}\%$ of 408 acres of land at \$40 an acre?

13. What is the value of 24% of 1728 bushels of corn at \$.50 a bushel?

14. Mr. White is worth 35% of \$24,000 and Mr. Brown is worth 68% of \$20,500. How much more is Mr. Brown worth than Mr. White?

15. A man who was worth \$18,930 bequeathed $33\frac{1}{3}\%$ of his property to his son and $16\frac{2}{3}\%$ of it to his daughter. How much did each receive?

16. A man whose income is \$1500 a year pays 16% of it for rent, 20% of it for household expenses, and 19% of it for all other expenses. How much does he save in a year?

275. To find the rate when the base and percentage are given.

1. What part of 10 is 5? How many hundredths of 10 is 5? What per cent of 10 is 5?

2. What part of 30 is 3? How many hundredths of 30 is 3? What per cent of 30 is 3?

3. What part of 40 is 10? What per cent of 40 is 10?

4. What per cent of 20 is 4? Of 100 is 70? Of 80 is 40?

What per cent of:

5. 16 is 8?

10. 36 lb. is 12 lb?

15. $\frac{2}{3}$ is $\frac{1}{3}$?

6. 24 is 6?

11. 48 qt. is 8 qt.?

16. $\frac{3}{4}$ is $\frac{1}{4}$?

7. 60 is 12?

12. 60 ft. is 15 ft.?

17. $\frac{4}{5}$ is $\frac{2}{5}$?

8. 50 is 5?

13. 25 bu. is 10 bu.?

18. $\frac{5}{8}$ is $\frac{4}{8}$?

9. 80 is 4?

14. 24 mi. is 18 mi.?

19. $\frac{8}{15}$ is $\frac{2}{15}$?

20. Mary had 60 cents and spent 15 cents for some ribbon. What per cent of her money did she spend?

21. In a school of 80 pupils 20 are boys. What per cent of the pupils are boys ?

22. A man who had a farm of 120 acres sold 30 acres. What per cent of the farm did he have left ?

WRITTEN EXERCISES.

276. 1. What per cent of 300 acres are 60 acres ?

60 acres = $\frac{60}{300}$ of 300 acres

= $\frac{1}{5}$ of 300 acres

= $\frac{20}{100}$ of 300 acres ;

\therefore 60 acres are 20% of 300 acres

Or,

60 acres \div 300 acres = .20 = 20%.

EXPLANATION. — 60 acres are $\frac{60}{300}$, or $\frac{1}{5}$, of 300 acres.

$\frac{1}{5}$ expressed as hundredths is equal to 20 hundredths ; therefore, 60 acres are .20, or 20% of 300 acres.

Or, since the percentage is the product of the base and the rate, if the percentage is divided by the base the result is the rate. Therefore, the rate is $60 \div 300$, or .20, or 20 %.

277. Hence it is evident that $P \div B = R$.

What per cent of :

2. 900 tons are 450 tons ?

6. \$ 300 are \$ 72 ?

3. 400 gal. are 120 gal. ?

7. \$ 192 are \$ 48 ?

4. 800 rods are 500 rods ?

8. $\frac{10}{11}$ are $\frac{5}{11}$?

5. 640 miles are 160 miles ?

9. $\frac{5}{8}$ are $\frac{3}{8}$?

10. A man who has an annual income of \$ 1200 spends \$ 900 of it. What per cent of it does he spend ?

11. Helen read 70 pages of a book containing 280 pages. What per cent of the book did she read ?

12. A boy who had 150 marbles lost 60. What per cent of the whole number did he have left ?

13. A man who owed \$ 1200 paid 30% of the debt at one time and 20% of the remainder at another. What per cent of the debt did he then owe ?

14. A and B own a mill worth \$ 10,360. If A's share is worth \$ 4662, what per cent of the mill does B own ?

15. If 4 gallons of water are mixed with 76 gallons of vinegar, what per cent of the mixture is each?

16. A farmer raised 480 bushels of grain, of which 25% was wheat, $\frac{2}{3}$ was oats, and the rest was corn. How many bushels of each kind of grain did he raise, and what per cent of his whole crop was corn?

17. A regiment of 1400 men engaged in battle and lost in killed 70 men, in wounded 224 men, and 378 men were taken prisoners. What per cent of the regiment was killed? What per cent was wounded? What per cent was captured? What per cent was left for service?

278. To find the base when the percentage and rate are given.

1. Of what number of men are 10 men 50%, or $\frac{50}{100}$, or $\frac{1}{2}$?
2. Of what number of sheep are 20 sheep 25%, or $\frac{25}{100}$, or $\frac{1}{4}$?
3. Of what number of bushels are 80 bushels 20%, or $\frac{20}{100}$, or $\frac{1}{5}$?
4. Of what number of eggs are 30 eggs 60%, or $\frac{60}{100}$, or $\frac{3}{5}$?

Find the number of dollars of which:

- | | | |
|-----------------|-------------------|--------------------------------|
| 5. \$30 is 30%. | 10. \$24 is 8%. | 15. \$5 is $\frac{1}{2}$ %. |
| 6. \$80 is 10%. | 11. \$96 is 20%. | 16. \$10 is $12\frac{1}{2}$ %. |
| 7. \$15 is 5%. | 12. \$36 is 18%. | 17. \$30 is $33\frac{1}{3}$ %. |
| 8. \$75 is 1%. | 13. \$160 is 32%. | 18. \$20 is $16\frac{2}{3}$ %. |
| 9. \$35 is 7%. | 14. \$125 is 25%. | 19. \$50 is $66\frac{2}{3}$ %. |
20. Jane broke 4 eggs, or 10% of all the eggs she had in her basket. How many eggs had she?
21. George gave away 12 chestnuts, or 20% of all he had. How many did he have?
22. A man drew some money from the bank and paid 25% of it for a cow that cost him \$30. How much money did he draw from the bank?

WRITTEN EXERCISES.

279. 1. Find the number of books of which 300 books are 75%.

75%, or $\frac{3}{4}$ = 300 books.

$\frac{1}{4}$ = 100 books.

$\frac{1}{4}$, or whole = 400 books.

Or,

300 books $\div .75$ = 400 books.

EXPLANATION. — Since 75%, or $\frac{3}{4}$, or $\frac{75}{100}$, or $\frac{3}{4}$ of the number of books is 300, $\frac{1}{4}$ of the number of books is $\frac{1}{3}$ of 300, or 100, and 4 times 100, or 400, is the whole number of books.

Or, since the percentage is the product of the base multiplied by the rate, if the percentage is divided by the rate the quotient is the base. Therefore, 300 books $\div .75$, or 400 books, is the entire number of books.

280. Hence it is evident that $P \div R = B$.

Find the number of which:

2. 312 is 20%. 6. 112 is 16%. 10. 315 is $37\frac{1}{2}\%$.

3. 350 is 25%. 7. 238 is 28%. 11. 444 is $66\frac{2}{3}\%$.

4. 624 is 30%. 8. 315 is 42%. 12. 12 is $\frac{3}{4}\%$.

5. 225 is 15%. 9. 129.6 is 18%. 13. 4 is $1\frac{1}{3}\%$.

14. A man expended \$25 for an overcoat, which was 20% of the money he had. How much money had he?

15. A merchant's expenses were \$50 per day. If his expenses were 25% of his income, what was his income per day?

16. I paid 30% of my money for a carriage which cost \$165. How much money did I have?

17. A farmer lost 150 sheep, which was 75% of his flock. How many sheep had he originally?

18. A horse was sold for \$240, which was 80% of what he cost. How much did he cost?

19. A house was sold for \$4050, which was $112\frac{1}{2}\%$ of the cost. How much did the house cost?

20. A farmer sold 350 bushels of grain and had 30% of his crop left. How many bushels of grain did he raise?

PROFIT AND LOSS.

281. 1. A watch that cost \$100 was sold at a profit of 6% of the cost. How much was gained?

2. An article that cost \$100 was sold at a loss of 25% of the cost. How much was lost?

3. A horse that cost \$200 was sold at a gain of 15% of the cost. How much was gained? For how much was the horse sold?

4. If a piano that cost \$600 was sold at a loss of 10% of the cost, what was the loss? What was the selling price of the piano?

5. If a cow that cost \$40 is sold for \$50, how much is gained? What part of the cost is the gain? How many hundredths? What per cent of the cost is the gain?

6. Coal that cost \$5 a ton was sold for \$6 a ton. What per cent of the cost was the gain?

7. Cloth that cost \$4 a yard became damaged so that it had to be sold for \$3 a yard. What per cent of the cost was the loss?

8. Butter that cost 36 cents a pound was sold at a loss of 12 cents a pound. What per cent of the cost was the loss?

9. By selling tea at a gain of 10 cents a pound, a grocer gained 25%, or $\frac{1}{4}$, of the cost of a pound. What was the cost?

10. A farm was sold at a loss of 20% of the cost. If the loss on an acre was \$12, what did the farm cost per acre?

11. A carriage was sold for \$20 more than it cost. If the profit was 20% of the cost, what was the cost? What was the selling price?

282. Hence it is seen that:

Gain or loss is reckoned at a certain per cent of the cost, or sum invested.

WRITTEN EXERCISES.

283. 1. A hardware merchant makes a profit of 32% on lawn mowers. How much does he gain on a lawn mower that cost him \$8?

2. Find the profit on a farm that cost \$2150 and was sold at a gain of $12\frac{1}{2}\%$.

3. What was the selling price of a flock of sheep bought for \$1250 and sold at a loss of 8%?

4. Flour that cost \$3.50 a barrel was sold so as to gain 28 cents on a barrel. What was the gain per cent?

5. What is the loss per cent on goods bought for \$6450 and sold for \$5547?

6. A house and lot that cost \$4260 was sold for \$4899. What was the gain per cent?

7. If a grocer makes an average profit of $12\frac{1}{2}\%$ on his goods, how much does he pay for goods in a year if he gains \$1600 in a year?

8. What is the cost of corn sold at a loss of 5%, or 3 cents a bushel?

9. What does a dealer pay for hops, if he gains 15%, or $4\frac{1}{2}$ cents a pound?

10. What per cent is gained, if potatoes that cost 42 cents per bushel are sold for 49 cents per bushel?

11. A grain dealer bought a car load of wheat at 80 cents a bushel and sold it at a gain of 10%. What was his gain, if there were 450 bushels in the car?

12. If a quantity of wool was bought for \$1480 and sold for \$1258, what was the loss per cent?

13. If a quantity of lumber was bought for \$1125 and sold at a gain of $33\frac{1}{3}\%$, for how much was it sold?

14. A bicycle dealer made a profit of 40%, or \$14, on each bicycle. Find the selling price.

15. A steamboat that cost \$88,450 was sold at a loss of 14%. How much was received for it?

16. A merchant bought 420 yards of cotton at 6 cents a yard and sold it for \$31.50. What per cent did he gain?

17. A watch was sold for \$4.80 less than cost, which was at a loss of $8\frac{1}{3}\%$. How much did the watch cost?

18. A man bought a house for \$3500, and, after spending for improvements a sum equal to 40% of the cost, he sold it for \$6468. What per cent did he gain on the entire cost?

19. A man bought an automobile for \$848, but, owing to a decline in the market value, he was obliged to sell it at a loss of $37\frac{1}{2}\%$. How much did he receive for it?

20. The gain on a quantity of coal was \$921.90, which was a profit of 21%. What was the cost of the coal?

21. A grain dealer gained \$1625 by selling 2500 bushels of oats at a profit of $16\frac{1}{4}\%$. How much did the oats cost per bushel?

22. What per cent is gained, if $\frac{5}{8}$ of a stock of goods that cost \$12,000 is sold for what the whole cost?

23. A merchant bought goods for \$2400 and marked them to sell at a gain of 15%, but afterwards raised the price 5% of the marked price. How much did he receive for the goods?

24. A mill was sold at a gain of \$1017, or 15% of the cost. What per cent would have been lost if it had been sold for \$5966.40?

25. A man bought a horse for \$150 and a carriage for $133\frac{1}{3}\%$ of the cost of the horse. He sold the horse at a gain of 30% and the carriage at a gain of $12\frac{1}{2}\%$. What per cent was gained on the combined sales?

COMMISSION.

284. 1. My agent sold a quantity of flour for \$6000, If he charged 1% of the selling price for making the sale, how much did he receive for his services?

2. An agent bought a quantity of broadcloth for \$4000. If he charged 2% of the cost of the goods for making the purchase, how much did he receive?

3. An agent sold a quantity of wool for \$2000 and received 4% of the selling price for his services. How much did he receive; that is, how much was his *commission*?

4. An agent sold some property for \$1000, receiving 3% for his commission. How much did he receive? After taking out his commission, how much was left; that is, what were the *net proceeds*?

285. A person who buys or sells goods, or transacts business for another, is called a **Commission Merchant**, or **Agent**, or **Broker**.

286. The compensation allowed a commission merchant is called his **Commission**, or **Brokerage**.

287. The sum left after commission and other expenses have been paid is called the **Net Proceeds**.

288. It is evident that:

The commission is reckoned at a certain rate per cent upon the value of the sales or purchases.

WRITTEN EXERCISES.

289. 1. A commission merchant sold a quantity of apples for \$284, charging 5% commission. How much was his commission?

2. At $1\frac{1}{2}$ per cent, what was an agent's commission for selling \$2150 worth of sugar? What were the net proceeds of the sale?

3. A commission merchant sold a quantity of cotton for \$8245. What were the net proceeds, if the commission was 3%?

4. At $\frac{1}{4}\%$, what was my agent's commission for buying city lots valued at \$9200?

5. What is the commission at $2\frac{1}{2}\%$ for buying 400 yards of silk at \$1.10 a yard?

6. A real estate agent was paid \$120 for collecting rents to the amount of \$2400. What was his rate of commission?

7. A broker bought \$6000 worth of stock at a brokerage of $\frac{1}{8}\%$. What was the brokerage?

8. A man received \$2688 as the net proceeds of a sale of \$2800 worth of goods. Find the rate of commission allowed.

9. An agent charged $3\frac{1}{2}\%$ for buying goods. If his commission was \$149.80, how much did he pay for the goods?

10. My agent sold goods for me to the amount of \$2430. If he paid \$44 for cartage and other expenses and charged 2% commission, what were the net proceeds?

11. A dealer in hay forwarded to his agent 240 tons of hay that cost him \$12 a ton. If the hay was sold at \$16 a ton at a commission of $1\frac{3}{4}\%$, how much did the dealer gain?

12. An agent sold 4000 bushels of wheat and received 2% commission. If his commission amounted to \$104.80, how much did he receive per bushel for the wheat?

13. The net proceeds of a sale were \$401.10 and the commission was \$18.90. What was the rate of commission?

14. A commission merchant's sales amounted to \$225,000 during a certain year, and on this amount he averaged $2\frac{1}{2}\%$ commission. How much was his income that year?

15. Find the net proceeds of a sale of 240 barrels of flour at \$7.50 per barrel, if the agent charged a commission of $2\frac{1}{2}\%$, and paid \$20.50 freight charges.

COMMERCIAL DISCOUNT.

290. A manufacturer agrees to make a deduction, for cash, equal to 20% of the price of each article. If I pay him cash for a bill of goods which by *list prices* amounts to \$40, how much does he deduct from the bill; that is, what is the *commercial discount*?

How much must I pay for the goods; that is, what is the *net amount* of the bill?

291. A deduction from the price or value of anything is called a **Commercial Discount**.

292. Manufacturers and wholesale dealers issue *price lists*, from which *prices* they allow various discounts.

Sometimes several discounts are allowed the purchaser. In such cases the first discount is to be deducted; then the second is to be computed upon the remainder and deducted; and so on for each successive discount.

Thus, when the discounts allowed are 40%, 10%, and 5%, the 40% is first deducted; then from the remainder 10% is deducted; and from the last remainder 5% is deducted.

293. The amount of a bill less the discount is called the **Net Amount**.

WRITTEN EXERCISES.

294. Find the net amount of each of the following bills:

1. \$ 400, discounts 20% and 10%.

2. \$ 720, discounts 25% and 5%.

3. \$ 640, discounts 40% and 25%.

4. \$ 930, discounts $33\frac{1}{3}\%$ and 15%.

5. \$ 1000, discounts 10%, 10%, and 5%.

6. A bill of goods as listed amounted to \$800. What was due on the bill, discounts being 30% and 5%?

7. What is the difference, on a bill of \$500, between a discount of 20% and successive discounts of 10% and 10%?

INSURANCE.

295. 1. How much will it cost to secure myself against loss by fire, or to *insure* my buildings for \$ 10,000, if I pay an annual sum of 1% to those who take the risk ?

2. What will be the cost of insuring a building for \$ 5000 at $\frac{1}{2}\%$; that is, what will be the *premium* ?

3. What is the premium on goods worth \$ 16,000, if they are insured for $\frac{1}{2}$ of their value at $\frac{3}{4}\%$?

4. I paid an annual premium of \$ 80 for insuring my property at 2%. For how much was it insured ?

5. How much do I pay in 2 years for having my life insured for \$ 2000 at \$ 25 per \$ 1000.

296. Indemnity against loss or damage is called **Insurance**. Insurance is of two kinds, **Property Insurance** and **Personal Insurance**.

297. Indemnity against loss or damage by fire, or **Fire Insurance**, and against loss or damage by casualties at sea, or **Marine Insurance**, are some of the kinds of **Property Insurance**.

298. Indemnity against loss of life, or **Life Insurance**, and against injury occasioned by accident, or **Accident Insurance**, are varieties of **Personal Insurance**.

299. The contract between the insurance company and the person insured is called a **Policy**.

300. The policies issued by life insurance companies are of various kinds, the chief of which are the **Life Policy** and the **Endowment Policy**.

A policy that secures the payment of a sum of money at the death of the person insured is called a *Life Policy*.

A policy that secures the payment of a sum of money at a specified time, or at death if death occurs before the specified time, is called an *Endowment Policy*.

301. The sum paid for insurance is called the **Premium**.

WRITTEN EXERCISES.

302. 1. What is the annual premium on an insurance policy of \$2640 at $1\frac{1}{2}\%$?

2. What is the annual premium on a life insurance policy for \$5000 at \$27.60 per \$1000?

3. What is the premium for 3 years at $2\frac{2}{3}\%$ per year on a fire insurance of \$4300?

4. How much premium do I pay in 6 years on a life policy of \$4000 at \$28.20 per \$1000?

5. How much will it cost to insure a cargo of grain for \$30,000 at \$1.25 per \$100?

6. My house which is worth \$6000 is insured for $\frac{3}{4}$ of its value at $\frac{3}{4}\%$. How much premium do I pay in 5 years?

7. A mill worth \$12,000 was insured for $\frac{3}{4}$ of its value at $\frac{1}{8}\%$. What was the annual premium?

8. If an annual premium of \$90 is paid for an insurance of \$12,000, what is the rate of insurance?

9. The annual cost of a life insurance for \$8000 is \$236. What is the annual premium per \$1000?

10. If \$150 is paid for insuring property for 4 years at $\frac{5}{8}\%$, for how much is the property insured?

11. If a man pays \$30 for insuring his household furniture for $\frac{1}{2}$ of its value at $\frac{3}{4}\%$, what is the value of his furniture?

12. A miller insures 600 barrels of flour for $\frac{3}{4}$ of its value at $2\frac{1}{4}\%$, paying a premium of \$54. How much is the flour worth per barrel?

13. A man who has a twenty-year endowment policy for \$3000 pays an annual premium of \$30.70 per \$1000. At the expiration of the 20 years how much more will he receive from the company than he has paid to it in premiums, disregarding dividends and interest?

INTEREST.

303. When a person borrows money, he usually has to pay a certain *per cent* of the sum for the use of it for each year.

304. 1. When a sum equal to 6% of the money loaned is paid for the use of it for each year, how much is paid for the use of \$ 100 for 1 yr. or what is the *interest* for 1 yr.?

2. When the rate of interest is 5% per year, how much will be the interest of \$ 100 for 1 yr.? For 2 yr.? For 3 yr.?

3. When the rate of interest is 8% per year, what will be the interest of \$ 100 for 1 yr.? Of \$ 200 for 1 yr.? Of \$ 500 for 1 yr.?

4. When the rate of interest is 6% per year, what will be the interest of \$ 400 for 1 yr.? For 2 yr.? For $2\frac{1}{2}$ yr.? For $\frac{1}{2}$ yr.?

5. When the rate of interest is 4% per year, what will be the interest of \$ 200 for 2 yr.?

What will be the sum of \$ 200, the *principal*, and the interest; that is, what will be the *amount*?

6. When \$ 400 is loaned for 3 yr. at 5% per year, how much is the interest? What is the amount?

What is the amount of \$ 300 for 3 yr. at 3%? For 2 yr. at 4%? For 4 yr. at 2%?

What is the interest of:

7. \$ 600 for 1 yr. at 5%?

9. \$ 800 for 2 yr. at 4%?

8. \$ 500 for 1 yr. at 6%?

10. \$ 400 for 3 yr. at 5%?

11. \$600 for $1\frac{1}{2}$ yr. at 3%? 15. \$800 for $1\frac{1}{2}$ mo. at 6%?
 12. \$200 for $2\frac{1}{2}$ yr. at 5%? 16. \$600 for 1 mo. 10 da. at 6%?
 13. \$800 for $\frac{1}{2}$ yr. at 6%? 17. \$600 for 2 mo. 10 da. at 6%?
 14. \$600 for 1 mo. at 6%? 18. \$800 for 3 yr. at 5%?

305. The sum paid for the use of money is called **Interest**.

306. The sum for the use of which interest is paid is called the **Principal**.

307. The sum of the principal and interest is called the **Amount**.

308. In computing interest it is usual to regard a year as 12 months, and a month as 30 days.

WRITTEN EXERCISES.

- 309.** 1. What is the interest of \$420.50 for 5 yr. at 5%?

SOLUTION.

\$420.50 Principal.

.05 Rate.

\$21.0250 Interest for 1 yr.

5

\$105.1250 Interest for 5 yr.*

Find the interest of:

2. \$212.34 for 3 yr. at 4%. 5. \$219.20 for 4 yr. at 7%.
 3. \$426.12 for 4 yr. at 5%. 6. \$324.16 for 5 yr. at 4%.
 4. \$324.25 for 5 yr. at 6%. 7. \$582.30 for 6 yr. at 5%.
 8. What is the interest of \$366.20 for 3 yr. 8 mo. at 6%?

SOLUTION.

\$366.20

.06

\$21.9720, int. for 1 yr.

3

\$65.9160, int. for 3 yr.

$\frac{1}{2}$ of the int. for 1 yr. = 10.9860, int. for 6 mo.

$\frac{1}{2}$ of the int. for 6 mo. = 3.6620, int. for 2 mo.

\$80.5640, int. for 3 yr. 8 mo.

Find the interest of:

9. \$412.20 for 2 yr. 8 mo. at 4%.
10. \$318.36 for 3 yr. 6 mo. at 6%.
11. \$425.24 for 4 yr. 5 mo. at 5%.
12. \$685.60 for 3 yr. 7 mo. at 7%.
13. \$437.28 for 5 yr. 2 mo. at 8%.
14. \$314.26 for 4 yr. 4 mo. at 3%.
15. \$736.16 for 2 yr. 10 mo. at 6%.

16. Find the amount of \$524.36 for 2 yr. 5 mo. 18 da. at 7%.

SOLUTION.

\$524.36

.07

\$36.7052, int. for 1 yr.

2

\$73.4104, int. for 2 yr.

$\frac{1}{4}$ of the int. for 1 yr. =	12.2350, int. for	4 mo.
$\frac{1}{4}$ of the int. for 4 mo. =	3.0587, int. for	1 mo.
$\frac{1}{4}$ of the int. for 1 mo. =	1.5293, int. for	15 da.
$\frac{1}{4}$ of the int. for 15 da. =	.3058, int. for	3 da.

\$90.5392, int. for 2 yr. 5 mo. 18 da.

524.36 , principal.

\$614.8992, amount.

Find the interest and amount of:

17. \$360.24 for 2 yr. 7 mo. 15 da. at 5%.
18. \$415.48 for 3 yr. 5 mo. 10 da. at 6%.
19. \$217.36 for 2 yr. 6 mo. 15 da. at 4%.
20. \$423.35 for 3 yr. 8 mo. 15 da. at 6%.
21. \$720.16 for 3 yr. 7 mo. 17 da. at 5%.
22. \$468.36 for 3 yr. 9 mo. 10 da. at 4%.
23. \$219.47 for 2 yr. 5 mo. 12 da. at 6%.
24. \$325.34 for 1 yr. 1 mo. 13 da. at 7%.
25. \$436.33 for 4 yr. 2 mo. 10 da. at 5%.
26. \$297.45 for 5 yr. 3 mo. 15 da. at 6%.

310. The six per cent method.

1. What is the interest of \$ 1 at 6% for 1 yr.? For 1 mo., or $\frac{1}{12}$ of a year? For 2 mo.? For 4 mo.? For 5 mo.?

2. What is the interest of \$ 1 at 6% for 6 da., or $\frac{1}{2}$ of a month? For 12 da.? For 18 da.? For 24 da.?

3. What is the interest of \$ 1 at 6% for 1 da., or $\frac{1}{365}$ of 6 da.? For 2 da.? For 3 da.? For 5 da.?

4. Since the interest of \$ 1 at 6% for 1 yr. is \$.06, for 1 mo. is \$.005, for 6 da. is \$.001, and for 1 da. is \$.000 $\frac{1}{6}$, what is the interest of \$ 1 at 6% for 1 yr. 1 mo. 7 da. (6 da. + 1 da.)? For 2 yr. 4 mo. 12 da.?

5. What is the interest of \$ 1 at 6% for 2 yr. 6 mo. 12 da.? Of \$ 100? Of \$ 200? Of \$ 300?

WRITTEN EXERCISES.

311. 1. What is the interest of \$ 240.50 for 2 yr. 4 mo. 12 da. at 6%?

SOLUTION.

The interest of \$ 1 for 2 yr.	= \$.12
The interest of \$ 1 for 4 mo.	= .02
The interest of \$ 1 for 12 da.	= <u>.002</u>
The interest of \$ 1 for 2 yr. 4 mo. 12 da.	= .142
The interest of \$ 240.50 = \$.142 \times 240.50, or \$ 34.151.	

What is the interest of:

2. \$ 480.25 for 1 yr. 8 mo. 24 da. at 6%?

3. \$ 360.10 for 2 yr. 6 mo. 18 da. at 6%?

4. \$ 542.80 for 3 yr. 2 mo. 6 da. at 6%?

5. \$ 920.20 for 2 yr. 5 mo. 9 da. at 6%?

6. \$ 624.40 for 4 yr. 7 mo. 15 da. at 6%?

7. \$ 750.50 for 3 yr. 2 mo. 12 da. at 7%?

SUGGESTION. — Find $\frac{7}{6}$ of the interest of \$ 750.50 for 3 yr. 2 mo. 12 da. at 6%; the result will be the interest at 7%.

8. \$ 864.60 for 2 yr. 4 mo. 15 da. at 5%?

9. \$ 1236 for 1 yr. 6 mo. 24 da. at 4%?

PROMISSORY NOTES.

312. A written promise to pay a sum of money at a specified time is called a **Promissory Note**, or simply a **Note**.

313. The person who makes the promise, or who signs the note, is the **Maker**, or **Drawer**.

314. The person to whom the note is made payable is the **Payee**.

315. The person who owns the note is the **Holder**.

316. The sum to be paid is the **Face** of the note.

317. The following are some *forms* of notes:

\$ 580.44.

ALBANY, N.Y., July 16, 1900.

Three months after date, I promise to pay George W. Stevenson, or order, Five Hundred Eighty $\frac{44}{100}$ dollars, for value received, with interest.

WILMUR H. FITCH.

\$ 2000.

SAN FRANCISCO, CAL., August 15, 1900.

For value received, on demand, I promise to pay Edward Griffith, or bearer, Two Thousand Dollars.

HARVEY D. BROWN.

318. A note should contain the name of the place where it is made, the date, the time when payable, the face of the note written in words, and the words "for value received," also the words "with interest," if such be the contract.

1. When no rate of interest is specified, the *legal rate*, or the rate authorized by law in the place where the note is made, is always understood.

2. If no place of payment is named in the note, it is payable at the maker's place of business.

3. If the words "with interest" are omitted, the note will not draw interest until it is due.

319. The person who writes his name across the back of a note to transfer it to another person, or to guarantee its payment, is called an **Indorser**.

The payee may indorse by simply writing his *name* on the back of the note. It is then payable to the holder or bearer. He may write, "Pay to A—— B——." It is then payable to A—— B—— only.

He may also write, "Pay to A—— B——, or order," and it is then payable to any person to whom A—— B—— may order it paid.

There are also other forms of indorsement.

320. A note that may be sold or transferred is a **Negotiable Note**.

To be negotiable, a note must be payable to the order of the *payee* or to the *bearer*.

Thus, both of the foregoing notes are negotiable.

321. A note that is payable to the *payee only* is a **Non-Negotiable Note**.

Thus, if the words "or order" and "or bearer" were omitted from the above notes, the notes would be non-negotiable.

322. A note is *payable* at the time specified, but *three days of grace* are allowed in some states before the note is said to *mature*. It is then *legally* due.

Days of grace have been abolished in New York, New Jersey, Pennsylvania, Connecticut, Vermont, Missouri, Massachusetts, and several other states.

WRITTEN EXERCISES.

323. Write the following notes, dating them when and where they are written:

1. A negotiable note for \$400, making H. H. Sours the payee and G. A. Smith the maker, interest at 5%.

2. Two negotiable notes, differing in form, for \$3000, due in four months with interest, payable to H. S. Dudley.

3. Indorse the notes in the preceding exercise to transfer one to the bearer and the other to S. H. Bruce.

4. A non-negotiable note for \$715.40, payable on demand, without interest, making Samuel G. Harmon the payee and J. J. Barber the drawer.

5. A note from the following data: face, \$600; negotiable; maker, Robert Clark; payee, A. C. Salls; payable in six months; interest at the legal rate.

BANK DISCOUNT.

324. An institution, chartered under the law, to receive money for safe keeping, to loan money, or to issue notes or bills to circulate as money is a **Bank**.

325. A considerable part of the business of most banks is the paying of notes before they are due.

If the banker becomes satisfied that the note is valid, or properly secured by indorsement, he may advance the sum due less the simple interest for the time the note has yet to run.

326. The number of days from the time a note is discounted to the time when it legally matures is the **Term of Discount**.

327. Simple interest collected in advance upon the sum due on a note at its maturity is called **Bank Discount**.

Banks usually demand that notes which they discount be made payable at the bank.

328. The sum due on a note at its maturity, less the bank discount, is called the **Proceeds**, or **Avails**, of the note.

WRITTEN EXERCISES.

329. 1. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a note for \$500, dated July 15, 1900, for three months, without interest or days of grace, and discounted Sept. 3, 1900, at 6%.

SOLUTION.

The note *matures* 3 mo. after July 15, or Oct. 15.

The *term of discount* is from Sept. 3 to Oct. 15, 42 da.

The *bank discount* is the interest of \$500 at 6% for 42 da., which is \$3.50.

The *proceeds* = \$500 - \$3.50 = \$496.50.

NOTE. — Although the time in a note may be expressed in *months*, the term of discount at a bank is usually reckoned by counting the actual number of days from the date of discount to the date of maturity, 360 days being regarded as a year.

2. Solve the preceding exercise when days of grace are allowed and the note bears interest at 6%.

SOLUTION.

The note *matures* 3 mo. 3 da. after July 15, or Oct. 18.

The *term of discount* is from Sept. 3 to Oct. 18, 45 da.

The sum due on the note at its maturity is \$500 plus the interest on that sum for 3 mo. 3 da. at 6%, or \$507.75.

Then, the *bank discount* is the interest of \$507.75 at 6% for 45 da., which is \$3.808+.

The *proceeds* = \$507.75 - \$3.808+ = \$503.942-.

NOTE. — Most notes discounted at a bank do not draw interest; but when interest-bearing notes are offered for discount, the discount is computed upon the amount of the note at its maturity.

Find the bank discount and proceeds of the following notes, no days of grace being allowed:

3. Face, \$300; term of discount, 2 mo. 15 da.; rate of discount, 6%.

4. Face, \$580; term of discount, 3 mo. 18 da.; rate of discount, 5%.

5. Face, \$640; date, Sept. 20, 1900; time, 2 mo.; date of discount, Oct. 3, 1900; rate of discount, 4%.

6. Face, \$800; date, Dec. 10, 1900; time, 3 mo.; date of discount, Jan. 21, 1901; rate of discount, 6%.

7. Face, \$1000; date, Mar. 7, 1900; time, 4 mo.; date of discount, Apr. 26, 1900; rate of discount, 8%; the note bears interest at 6%.

Find the bank discount and proceeds of the following notes, days of grace being allowed:

8. Face, \$560; date, Apr. 24, 1899; time, 2 mo.; date of discount, May 22, 1899; rate of discount, 6%.

9. Face, \$1200; date, May 14, 1898; time, 4 mo.; date of discount, July 25, 1898; rate of discount, 6%.

10. Face, \$1600; date, June 17, 1900; time, 3 mo.; date of discount, July 16, 1900; rate of discount, 6%; the note bears interest at 7%.

GENERAL REVIEW.

330. 1. What will 50 sheep cost if 75 are worth \$375?
2. How many years is it since the Declaration of Independence was signed?
3. A rope 13 yd. 2 ft. 6 in. long was cut into 3 equal parts. How long were they?
4. A farmer sold at one time 3 bu. 3 pk. 7 qt. of clover seed, and at another 5 bu. 2 pk. 6 qt. How much did he sell?
5. A wagon wheel is 15 ft. 8 in. in circumference. How many revolutions will it make in going 5 miles?
6. James spent 57 cents, which was $\frac{3}{8}$ of all the money he had. How much money had he?
7. A father gave his children \$8 apiece. If he had given them \$12 apiece it would have taken \$40 more. How many children were there?
8. The larger of two fractions is $\frac{1}{2}$, and the difference between them is $\frac{1}{4}$. What is the smaller fraction?
9. If $\frac{3}{4}$ of an acre of land produces 125 bushels of potatoes, how many bushels will $3\frac{1}{2}$ acres produce?
10. A can do a piece of work in 10 days, and B can do it in 8 days. In what time can both do it?
11. A man who owned $\frac{2}{3}$ of a factory sold $\frac{1}{3}$ of his share for \$3500. What was the factory worth at that rate?
12. Two boys did some work for 65 cents. If one of them earned 15 cents more than the other, how much did each earn?

13. A man who spent $\frac{1}{4}$ of his money and \$12 more found that he had \$15 left. How much had he at first?

14. A man died leaving to his 3 children a farm of 317 A. 115 sq. rd. to be divided equally among them. What was each one's share of the land?

15. How many acres of land, at \$45 per acre, can I get for 90 oxen at \$37.75 per head?

16. Mr. David Rounds purchased from D. Henderson & Co. 13 yd. muslin @ 8¢, 8 yd. sheeting @ \$.11 $\frac{1}{2}$, 25 yd. ribbon @ 7¢ per yd., 10 lb. cotton batting at 8 $\frac{1}{2}$ ¢, 2 pr. gloves @ \$1.25, 3 doz. pearl buttons @ \$.27. Make out a receipted bill.

17. How long will it take a person to count \$1,000,000 if he counts \$5 per second for 10 hours per day?

18. If a railroad train runs at the rate of 52 $\frac{1}{2}$ mi. per hour, how far will it run in 8 hr. 40 min.?

19. How many feet board measure are there in 45 joists, each 18 ft. long, 16 in. wide, and 2 in. thick?

20. A farmer sold 3 T. 18 cwt. 85 lb. of hay at \$9.50 per ton. How much did he get for it?

21. A cask of wine contained 58 gal. 3 qt. 1 pt. How much was it worth at \$2.75 per gallon?

22. A silversmith sold some silver plate at \$1.50 per ounce, receiving for it \$93. How much did he sell?

23. A merchant had 320 yd. of cloth, which he made into suits which required 6 yd. 2 ft. each. How many suits did he make?

24. What will be the expense of painting a gable roof 48 ft. long and 25 ft. wide at \$.30 per sq. yd.?

25. How many yards of matting, 1 $\frac{1}{2}$ yd. wide, will be required to cover a floor 7 yd. 2 ft. 7 $\frac{1}{2}$ in. wide, and 18 ft. long? What will be the cost of it at \$.70 per lineal yd.?

26. A man has a lot containing 10 acres that is 25 rods wide. How long is it?

27. A money-lender loaned \$ 520 for two years at 6%. How much money should he receive as principal and interest at the end of the time ?

28. One number is $\frac{3}{4}$ of another, and their sum is 490. What are the numbers ?

29. A man who owned $\frac{2}{3}$ of a vessel sold $\frac{1}{3}$ of his share for \$ 12,500. At that rate, what was the vessel worth ?

30. Two men engaged in business, one furnishing $\frac{3}{4}$ of their capital, and the other the rest. If they gained \$1477, what was each one's share of the gain ?

31. Mr. Davis loaned his nephew \$ 350 on Jan. 1, 1890, at 6% interest. If the debt was paid Jan. 1, 1893, how much was the amount ?

32. How much will it cost to carpet a room 18 ft. wide and 24 ft. long at \$ 1.15 per sq. yd. ?

33. A druggist sold 1 lb. of quinine, which was made into two-grain pills. How many pills were there ?

34. A stationer bought 10 reams of paper at \$1.90 per ream, and sold it at 1 cent per sheet. How much was his gain ?

35. Henry bought at a grocer's 15 lb. 12 oz. of butter at 32 cents per pound. How much did it cost him ?

36. Two brothers earned during their vacation \$ 110, but one of them earned only $\frac{5}{8}$ as much as the other. How much did each earn ?

37. If $\frac{3}{4}$ of a bbl. of flour costs \$4.20, what will 7 bbl. cost ?

38. Divide $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{9}$ of $\frac{10}{14}$ of $\frac{9}{11}$ by $\frac{1}{2}$ of $\frac{10}{12}$ of $\frac{7}{8}$ of $\frac{3}{4}$ of 9.

39. Sound travels about 1140 feet per second. If a flash of lightning is seen, but the thunder accompanying it is not heard until 35 seconds later, how far away is the lightning ?

40. The product of two numbers is 3.35, and one of them is .25. What is the other number ?

41. A city lot 60 ft. by 150 ft. was sold for \$500. At what price per acre was the land sold?

42. Divide .0036783 by 2.01 and write the quotient in words.

43. The sum of two numbers is 285, and their difference is $\frac{3}{11}$ of the greater number. Find the numbers.

44. A dealer bought plows at 20% and 10% off from list price. Find the cost of a plow listed at \$12.50.

45. What fractional part of a week is 2 da. 17 hr. 30 min.?

46. If the water from a salt spring yields $6\frac{1}{2}\%$ of its weight in salt, how many tons of the water will be required to make 1000 pounds of salt?

47. Find the net amount of a bill of goods billed at \$1800, the discounts being 30%, 20%, and 5%.

48. Find the total cost of 3250 lb. of stove coal at \$5.25 per ton and 100 lb. of cannel coal at \$6.90 per ton.

49. Find the proceeds of a note for \$186 given for 3 months and discounted at bank the day it was made, at 6%, no days of grace being allowed.

50. Reduce $.06\frac{2}{3}$ to a common fraction; to the form of per cent.

51. Of what number is 1.83 three eighths per cent?

52. If the bricks in a building cost \$701.50 at \$7.625 per M., how many bricks were purchased?

53. When the Empire State Express is running 62 miles an hour, how long will it be in passing the space between 32 telegraph poles set 66 yards apart?

54. A merchant has $\frac{2}{3}$ of his capital invested in goods, $\frac{1}{4}$ in land, $\frac{2}{15}$ in stock, and the rest, which is \$619.70, deposited in bank. What is the amount of his capital?

55. Find the proceeds of a 4-month note for \$300 made July 19, 1900, and discounted the day it was made at the Commercial National Bank of Albany, N. Y., at 6%.

56. Find the exact number of days from Jan. 17, 1899, to June 3, of the same year.

57. Multiply 5 rd. 1 yd. 2 ft. by $10\frac{1}{2}$.

58. If I divide my farm of 207 A. $93\frac{1}{2}$ sq. rd. into 18 equal fields, how much land will there be in each field?

59. Reduce each of the following to yards and find the sum of the results: $\frac{3}{4}$ rd., $2\frac{2}{3}$ ft., and 21 in.

60. A dealer mixed teas that cost 28 cents, 35 cents, and 37 cents per pound, respectively, in equal quantities, and sold the mixture at an advance of 20% on the cost. What was the selling price per pound?

61. The premium on a policy of insurance at $\frac{3}{4}\%$ is \$27.37 $\frac{1}{2}$. Find the amount of the policy.

62. Find the interest on \$196 for 9 mo. 1 da., at $3\frac{1}{2}\%$.

63. A triangular lot whose area is one acre has an altitude of 16 rd. How long is its base?

64. Write a non-negotiable note for \$850.50 payable on demand, making R. D. Grover the payee and O. H. King the maker.

65. A merchant insures a cargo worth \$12,000 for $\frac{5}{8}$ of its value at $3\frac{1}{2}\%$. If the ship should be wrecked, what would be his loss in goods and premium paid?

66. How many yards of carpeting $\frac{3}{4}$ yd. wide will be required to carpet a room 12 ft. by 13 ft. 6 in., the strips running crosswise? Running lengthwise?

67. A commission merchant sold potatoes to the amount of \$582.60. After paying \$51.25 for expenses, and deducting his commission of 5%, what were the net proceeds?

68. If a man earns \$450 every $2\frac{1}{2}$ months, and spends in 7 months as much as he earns in 4 months, how much does he save in a year?

69. Find the proceeds of a bank note for \$275, given for 2 months, and discounted the day it was made, at 6%, allowing days of grace.

70. A farmer delivered at a warehouse four loads of wheat weighing respectively 3313 lb., 3510 lb., 3470 lb., and 3291 lb. How much should he have received at \$ 1.10 per bu., if 1 bu. weighs 60 lb. ?

71. Reduce .36225 bu. to quarts.

72. Required the cost of 35 pieces of scantling 18 ft. long, 4 in. wide, and 2 in. thick, at \$ 14 per M. board measure.

73. The product of four numbers is 610.74, and three of the numbers are respectively 24, .9, 3.25. Find the other number.

74. The cost of a load of coal weighing 2370 lb. was \$ 7.11. Required the price per ton.

75. How many cords of wood are there in a pile of four-foot wood 16 ft. long and $5\frac{1}{2}$ ft. high ?

76. A house and lot that cost \$ 8436 were sold for \$ 7030. What was the loss per cent ?

77. Find the interest on \$ 820.44 from June 17, 1889, to April 11, 1892, at 4%.

78. What premium must be paid to insure a cargo of 4880 bu. of wheat valued at \$ 1.04 per bu. at $1\frac{1}{2}\%$, the policy being for only $\frac{5}{8}$ of the value ?

79. Reduce $3\frac{3}{4}\%$ to a decimal; to a common fraction.

80. A room 18 ft. by 16 ft. was carpeted with carpet $\frac{3}{4}$ of a yard wide so that the waste was the least possible. How many lineal yards of carpet were required ?

81. The difference between two numbers is 3.074 and the smaller number is .826. Find the larger number.

82. What will be the amount of \$ 348 on interest for 7 months at 3% ?

83. Divide 51 rd. 4 yd. 1 ft. 6 in. by 9.

84. An agent sold 26 copies of a book at \$ 3.50 per copy and was allowed 40% commission. Find the net proceeds of his sales.

85. What fraction of a rod is 3 ft. 8 in. ?
86. Find the agent's commission at 4 per cent on a sale of 860 barrels of apples at \$2.75 per barrel.
87. What fractional part of 22 lb. 8 oz. is 5 lb. 14 oz. ?
88. Find the interest and amount of \$222.50 for 1 yr. 5 mo. 6 da. at 6 per cent.
89. If a grocer sells for 36 ¢ per pound some coffee that cost him $26\frac{1}{4}$ ¢ per pound in New York and 25 ¢ a hundred for freight and cartage, what is his gain per cent ?
90. The list price of a carriage is \$160. What is the net price after deducting discounts of 25% and 10% ?
91. If the value of $\frac{1}{2}\frac{1}{2}$ of a piece of property is \$389.50, what is the value of the remainder ?
92. The sum of two numbers is 5 da. 8 hr. 24 min., and their difference is 22 hr. Find the greater number.
93. What per cent of 239 bu. is 14 bu. 3 pk. 6 qt. ?
94. Required the cost of two sticks of timber, each 18 ft. by 1 ft. 2 in. by 1 ft., at \$22.50 per M., board measure.
95. Reduce the fraction $\frac{333}{888}$ to its lowest terms.
96. What fractional part of a mile is 3 rd. 4 yd. 2 ft. 6 in. ?
97. A merchant closed out a stock of goods that cost \$432 at a loss of 28%. How much did he lose ?
98. An agent sold 1014 bushels of oats at 41 cents per bushel, paid \$33.74 freight charges, and retained $3\frac{1}{2}$ % commission. What were the net proceeds of the sale ?
99. How many acres of land are there in a straight piece of road $1\frac{1}{4}$ miles long and 4 rods wide ?
100. A cooper paid \$77.90 for 16,400 barrel staves. Required the price per M.
101. The running time of the Empire State Express from New York to Buffalo is 8 hr. 30 min., and the distance is 440 miles. If four stops of five minutes each are made, what is the average speed per hour ?

102. Required the cost of 735 fence boards, each 16 ft. long and 5 in. wide, at \$14.50 per M.

103. Reduce $.0048\frac{1}{2}$ to the form of per cent.

104. A merchant sold a quantity of goods for \$93.15, and thereby lost $\frac{7}{8}$ of what the goods cost him. Find the cost of the goods.

105. Required the cost to the purchaser of an article listed at \$54 but sold subject to commercial discounts of 25%, $16\frac{2}{3}\%$, and 5%.

106. If .276 of a piece of property is worth \$524.40, what is the value of the entire property?

107. What per cent of 6925 must be added to that number to make the sum 8310?

108. Reduce to common fractions in their simplest form (a) .06125; (b) $.54\frac{1}{8}$; (c) .06 of $.375$.

109. Mr. White started on a journey of $89\frac{7}{8}$ miles, and traveled $66\frac{1}{4}$ miles the first day. What part of the distance had he yet to travel?

110. If bell metal is composed of 78 parts of copper and 22 parts of tin, what weight of each will there be in a bell that weighs 450 lb.?

111. At \$19.25 per 1000 lb. for steel rails, what will be the cost of rails weighing 80 pounds to the yard for a mile of track?

112. If the annual premium paid for insuring a building is \$132, and the rate charged is $\frac{4}{5}\%$, for what sum was the building insured?

113. If the list price of a furnace sold at 30% and 10% off is \$240, what is the net price?

114. Lemons that cost \$.16 $\frac{2}{3}$ per dozen are sold for \$.35 per dozen. What is the gain per cent?

115. At $1\frac{1}{8}\%$, the premium on a policy covering three fourths of the value of a stock of merchandise amounts to \$189.84 $\frac{1}{2}$. Find the value of the entire stock.

116. Find the amount of \$86.75 on interest for 1 yr. 7 mo. 6 da. at 4%.

117. Reduce 2286 inches to rods and yards.

118. If I sell silk at an advance of $16\frac{2}{3}\%$ and gain \$.25 per yard, what is the selling price?

119. Reduce 2 lb. 12 oz. to a fraction of a hundredweight.

120. Reduce $\frac{2}{3}$ of 5 to a fractional part of 11.

121. Find the proceeds of a note for \$1350 discounted at a bank 4 mo. 5 da. before it was due, at 6%, no days of grace being allowed.

122. How many bushels of wheat will fill a bin 8 ft. by 4 ft. 2 in. by 3 ft. 8 in.?

123. Express $\frac{7}{8}\%$ in the form of a common fraction and in the form of a decimal.

124. A stock of goods valued at \$18,000 was insured for $\frac{3}{8}$ of its value at $1\frac{1}{2}\%$. Find the premium.

125. Find the cost of 16 planks each 16 ft. long, 14 in wide, and $2\frac{1}{4}$ in. thick, at \$23.50 per M. board measure.

126. How much is my agent's commission for buying 16,000 pounds of pork at $6\frac{1}{2}$ cents a pound, if he charges a commission of 5%?

127. If 5 men consume $36\frac{3}{4}$ lb. of bread in $12\frac{1}{4}$ days, in how many days will 11 men consume 44 lb.?

128. A stockman paid \$47.25 premium to insure 63 head of horses at $1\frac{1}{4}\%$. For how much was each horse insured?

129. At 25 cents a square yard, how much will it cost to plaster the walls and ceiling of a room 16 ft. by 14 ft. and 9 ft. 6 in. high, if $\frac{1}{16}$ of the whole surface is allowed for openings?

130. A farmer sold a grocer 70 bushels of potatoes at 18 cents per bushel and 8 barrels of apples at \$2.75 per barrel, receiving in payment \$20 and the balance in sugar at 5 cents per pound. How much sugar did he receive?

131. Reduce 7 gal. 3 qt. 1 pt. to a fraction of a barrel.

132. Divide .03125 by .000025.

133. How many cords of wood are there in a pile 32 ft. long, 6 ft. high, and 4 ft. wide?

134. A load of hay weighing 3750 lb. is sold for \$16.80 per ton. Find the value of the load.

135. Find the cost of filling a bin 18 ft. by 5 ft. 6 in. by 5 ft. 4 in. with coal at \$5.25 per ton, assuming that one ton occupies 40 cubic feet of space.

136. From 1 subtract $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$, divide the remainder by $5\frac{1}{8} - 4\frac{7}{8}$, and find the difference between 3 and the quotient.

137. How many gallons will a tank hold, if it is 10 ft. by 5 ft. 6 in. by 4 ft. 1 in.?

138. A boy buys apples at the rate of 5 for 2 cents and sells them at the rate of 4 for 3 cents. How many must he sell to gain 28 cents?

139. A man owning $\frac{3}{4}$ of a factory sold $\frac{2}{5}$ of his share to one man and $\frac{1}{5}$ of his share to another. What per cent of the factory did he still own?

140. A rectangular field is 40 rods by 28 rods. How many acres does it contain?

141. If $\frac{1}{3}$ of a man's property is real estate, $\frac{2}{10}$ of it mortgages, $\frac{1}{6}$ of it notes, and the remainder, \$1200, cash, what is the value of his entire property?

142. An agent's commission on a sale of produce was \$31.15. What rate of commission did he receive, if the net proceeds of the sale were \$591.85?

143. Find the proceeds of a note for \$350 given for three months and discounted at an Albany, N. Y., bank the day it was made, at 6%.

144. A bicycle party rode $75\frac{1}{2}$ miles at the average rate of $10\frac{1}{2}$ miles an hour. If they started at 8.53 A.M., at what hour did they complete the trip?

145. How many feet per second does a railroad train travel when running at the rate of 60 miles an hour?

146. What is the rate of commission when \$1656.72 is received as the net proceeds of a sale of 236 tons of hard coal at \$7.20 per ton?

147. What is the least common denominator to which the following fractions can be reduced: $\frac{5}{8}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{9}{14}$?

148. A car load of wheat weighs 5 T. 320 lb. How much is the wheat worth at 93 cents per bushel, if one bushel weighs 60 lb.?

149. Find 8% of 60 bu. 3 pk.

150. Three men did a piece of work for which they were paid \$54. If one worked 6 hr. per day for 11 days, another 9 hr. per day for 10 days, and the third 12 hr. per day for 5 days, how much should each receive?

151. Find the cost of a bicycle listed at \$100, but purchased at discounts of 30%, 10%, and 5%.

152. A note for \$500 and interest at 6% was given for one year, and after having run nine months was discounted at Buffalo, N. Y., at 6%. Find the proceeds.

153. A block of granite is 5 ft. 9 in. by 4 ft. 2 in. by 3 ft. 8 in. How much did it cost at \$3.15 per cubic yard?

154. A man bought a piece of land 30 rd. long and 60 rd. wide for \$200 per acre. He divided it into building lots each containing one fourth of an acre and sold them for \$150 each. How much did he gain?

155. Find the total area in square feet, of the ceiling and walls of a room 18 ft. 9 in. long, 14 ft. 8 in. wide, and 10 ft. 6 in. high.

156. A commission merchant sold 6000 baskets of grapes at $12\frac{3}{4}$ cents per basket and after deducting \$18.70 for transportation charges and his commission of 3% sent the net proceeds to the person for whom he sold the grapes. How much did he send?

157. Find the amount of \$312 on interest at 5% from July 5, 1898, to January 10, 1900.

158. What is the capacity, in gallons, of a tank 18 ft. 8 in. by 5 ft. 3 in. by 3 ft. 8 in.?

159. What is the date of maturity, term of discount, bank discount, and proceeds of a 2-month note for \$300, made and discounted July 10, 1900, days of grace being allowed, if the rate of discount is 6%?

160. A school room 36 ft. by 26 ft. by 11 ft. 6 in. has a seating capacity of 40 pupils. How many cubic feet of air are allowed for each pupil?

161. Required the cost of one dozen silver spoons, each weighing 18 pwt. 18 gr., at \$1.15 per ounce.

162. If it requires 12 days traveling 15 hours a day to make a journey of 3960 miles, how many hours a day will it require to make a journey of 1584 miles in 8 days, traveling at the same rate?

163. If my agent buys hay for \$8.50 per ton and charges a commission of 4%, how much does the hay cost me per ton?

164. A man rowed down a river at the rate of a mile in 12 minutes and returned at the rate of a mile in 20 minutes, the trip occupying 4 hr. 16 min. How far down did he go?

165. The gable of a barn is 27 feet wide and 9 feet high. Find its surface in square feet.

166. If I burn 2885 cubic feet of gas this month at \$1.50 per thousand, and am allowed 5% discount for prompt payment, how much will the gas for the month cost me?

167. A man purchased two adjoining pieces of land containing respectively 40 A. 50 sq. rd. and 110 A. 10 sq. rd. He sold from this tract $22\frac{1}{4}$ A. and 59.315 A. How many acres of land had he remaining?

168. What is the capacity, in bushels, of a bin 9 ft. 6 in. by 7 ft. 4 in. by 6 ft. 3 in.?

QUESTIONS FOR REVIEW.

331. NOTATION AND NUMERATION.—Define a unit; a number; notation. In what two ways may numbers be represented? Define numeration; Arabic notation. What characters are employed in the Arabic notation? How are numbers grouped in the Arabic notation?

When two figures are written side by side, what do they represent? Three figures? Four figures? Five figures? Six figures? Seven figures? How are numbers expressed by two figures read? By three figures? By four figures? By five figures? By six figures? By seven figures? How are the figures of a number often grouped for convenience in reading the number? Name five periods in succession, beginning at the right. Give a rule for pointing off numbers for reading. Which period may contain less than three figures?

How many cents are there in one dime? In one dollar? What places do cents occupy in the notation of United States money? What is used to separate dollars from cents? If the number of cents is less than 10, what figure is written in the first place to the right of the decimal point? What is the sign of dollars? Where is it written? What is a mill? In business, what is the rule about mills?

What is the Roman notation? What letters are employed in the Roman notation? Give the principles governing the Roman notation and illustrate each one.

ADDITION.—Define addition; sum, or amount. Describe the sign of addition; the sign of equality. How is + read?

How is $=$ read? How are numbers to be added arranged? Can they be added under any other arrangement? Why are units written under units, tens under tens, etc.? What is the most convenient way of adding several numbers?

When the units are added and the sum is greater than 9, what is done with the tens' figure?

SUBTRACTION. — Define subtraction; minuend; subtrahend; remainder. Which is the greater, the minuend or the subtrahend? How can the minuend be obtained from the subtrahend and the remainder? How can we find out whether the answer is correct? What is the sign of subtraction called? How is $10 - 4$ read? What is the meaning of $10 - 4$?

How are the minuend and subtrahend arranged for subtraction? Could the subtraction be performed if the subtrahend were written above the minuend? How is 8 subtracted from 24? 81 from 245? How is 25 subtracted from 100? What is done when some figure of the subtrahend expresses more than the corresponding figure of the minuend?

MULTIPLICATION. — Define multiplication; multiplicand; multiplier; product; factors of the product. Describe the sign of multiplication and its use. How can 3×4 be found by addition? Describe how 122 is multiplied by 3; 125 by 3; 142 by 3. Is it necessary to begin at the right to multiply? How is a number multiplied by 10? By 100? By 1000? By 1 with any number of ciphers annexed? By any number with ciphers annexed?

How is 136 multiplied by 67? How is a number multiplied by a number expressed by more than one significant figure? Give an explanation of the process. What are numbers called that are added to obtain the product? When there are ciphers between significant figures of the multiplier, how is the multiplication performed?

DIVISION. — Define division; dividend; divisor; quotient; remainder. Describe the sign of division. State how division may be indicated. How do we know that 4 is contained 5 times in 20? Solve an example in short division and explain the process. What divisors are usually employed in short division? Why are not longer divisors used?

How is a number most conveniently divided by 10, 100, 1000, etc.?

How does long division differ from short division? Give a rule for long division. How is the correctness of the result proved?

What is the use of the parenthesis, the vinculum, and the brackets? Illustrate their use. Give a rule for finding the value of such expressions.

FACTORING. — Define integer; factors of a number; exact divisor; prime number; composite number; prime factor; even number; odd number; factoring. How can we tell, without actually dividing, whether a number is divisible by 2? By 5? By 10? By 3? By 9? Describe the process of separating a number into its prime factors. Why may equal factors be canceled from the dividend and divisor? Why is cancellation useful? Make an example and solve it by cancellation.

FRACTIONS. — What is a fraction? Express 3 of the 5 equal parts of a unit. What are the terms of a fraction? What does each indicate? What is the difference between proper and improper fractions? Define and illustrate mixed number. What besides 3 of the 5 equal parts of 1 may the fraction $\frac{3}{5}$ express? Write in words the equivalent of $\frac{3}{5}$ in three ways.

Define reduction of fractions. Illustrate. How is a mixed number reduced to a fraction? What kind of a fraction results from this reduction? Why? What kind of fractions can be reduced to integers or mixed numbers? Why?

Illustrate. How is the reduction performed? How is a fraction reduced to lower terms? Illustrate. Give the reason for the process. When is a fraction in its lowest terms? How may a fraction be reduced to higher terms? What are similar fractions? Dissimilar fractions? Why are $\frac{2}{11}$ and $\frac{5}{11}$ called similar fractions? If several fractions have the same denominator, what is this denominator called? How are dissimilar fractions reduced to similar fractions having the least common denominator?

Give a rule for adding fractions so that the sum will be in as simple a form as possible. Give a rule for subtraction when one or both of the numbers are fractions. In what two ways may a fraction be multiplied by an integer? Illustrate each. How is an integer multiplied by a fraction? An integer by a mixed number? Multiply $\frac{3}{4}$ by $\frac{5}{7}$ and explain the process. Give the rule for multiplying a fraction by a fraction. How does the rule apply to $2\frac{1}{5} \times \frac{3}{4}$? To $5 \times \frac{3}{4}$? Multiply $\frac{3}{4}$ by $\frac{2}{3}$. What reduction of the result is avoided by cancellation? What is a compound fraction? How is a fraction divided by an integer? An integer by a fraction? An integer by a mixed number? Divide 2 by $\frac{3}{4}$ and explain the process. Explain how $\frac{2}{3}$ is divided by $\frac{3}{4}$. Give a rule for dividing a fraction by a fraction. Show that this rule has a general application in division, whether the dividend or divisor is a fraction, an integer, or a mixed number. What is a complex fraction?

How is the relation of one number to another found? What is an aliquot part of a number? Give some of the aliquot parts of a dollar. Find a number of which 25 is $\frac{5}{8}$ and explain how the whole of any number is found when a given part of it is known.

What is a decimal fraction? What is the meaning of the word decimal? What kind of a notation is the Arabic notation? How is the Arabic notation for integers extended

to express decimals? How is $\frac{7}{10}$ expressed as a decimal? $\frac{7}{100}$? $\frac{7}{1000}$? Give the rule for expressing such fractions in the decimal form. Name the first six orders of decimals. How many decimal places are required to express tenths? Hundredths? Thousandths? Ten-thousandths? Write as a decimal 16 tenths; 16 hundredths; 16 thousandths.

If ciphers are annexed to a decimal, how is the value of the decimal affected? How, if decimal ciphers are prefixed to a decimal? How does the number of places in a decimal compare with the number of ciphers in the denominator of the equivalent common fraction? Why? Explain that $.63 = .630 = .6300$. How are dissimilar decimals reduced to similar decimals? How is a decimal reduced to a common fraction? Illustrate. How is $\frac{5}{10}$ written as a decimal? Explain fully how $\frac{3}{8}$ is reduced to the decimal form. Give a rule for reducing a common fraction to a decimal.

Give a rule for adding decimals. Illustrate. Give a rule for subtracting one decimal from another. Illustrate. Show how this rule is applied when a decimal is subtracted from an integer. How does the number of decimal places in the product of two or more numbers compare with the number of decimal places in the factors? Illustrate. How is a decimal multiplied by 10? By 100? By 1000? Explain that $.5 \times .03 = .015$ and give a rule for multiplication of decimals. If the product does not contain a sufficient number of decimal places, what is done? How does the number of decimal places in the quotient compare with the number of decimal places in the dividend and divisor? Why? Give a rule for division of decimals. If the quotient does not contain a sufficient number of decimal places, what must be done? What may be done when the dividend contains fewer decimal places than the divisor? Illustrate. When the division is not exact, what is commonly done in business? How is a decimal divided by 10, 100, 1000, etc.?

ACCOUNTS AND BILLS. — What is an account? A bill? A receipted bill?

DENOMINATE NUMBERS. — Define and illustrate concrete number; abstract number; denominate number; simple denominate number; compound denominate number. Give the table of linear measures. Give several uses of this table.

What is a surface? An angle? A square? Illustrate each. What is a right angle? Draw one. How many feet are there in one yard? How many square feet are there in one square yard? How many inches are there in one foot? How many square inches are there in one square foot? How many yards are there in one rod? How many square yards are there in one square rod? How many rods are there in a mile? How many square rods are there in a square mile? Give the table for measures of surface, or square measures. Which is the larger, a tract of land six miles square, or a tract of land containing six square miles?

What is a solid? The volume of a solid? Illustrate. What kind of a solid is a cube? What are the bounding surfaces of a cube? Name three cubes that are used as units of measure. Give the table of cubic measures. Why are there 1728 cubic inches in one cubic foot and 27 cubic feet in one cubic yard? Name some articles that are measured by cubic measure.

What are liquid measures? How many gills are there in one pint? In one quart? What part of a pint is one gill? How many gallons are there in a barrel? What liquids are measured by the barrel? How many barrels and how many gallons are there in a hogshead? How many cubic inches are there in a gallon? What part of a cubic foot is a gallon? How many cubic feet are there in a barrel? Give the table of liquid measures.

What articles are measured by dry measure? Give the

table of dry measures. Which quart measure is the larger, that used to measure milk, or that used to measure beans? Prove it. What is the number of cubic inches in a bushel?

Give the table of avoirdupois weights. What measures are used in buying and selling coal? What part of a ton is a hundredweight? How many pounds are there in a ton? In a hundredweight? What part of a pound is one ounce? Four ounces? Give the table of troy weights. For what is troy weight used? What part of an ounce is one pennyweight? How many ounces are there in one pound avoirdupois? In one pound troy? How many grains are there in one pound avoirdupois? In one pound troy? What units of measure are commonly used in buying or selling the following: hay, gold, butter, meat, potatoes, berries, wheat, iron, milk, silver, livestock?

What units of measure are used by physicians and druggists? Give the table. How many grains of quinine are there in one pound? What part of a pound is a dram?

Give the table of time. How many minutes are there in one hour? In one day? How many seconds are there in one day? What part of a day is 12 hours? How many days are there in a week? In a year? In a leap year? Name the leap years from 1896 to 1921. How many years make a century? What centennial years are leap years? Name the months and give the number of days in each. Repeat the memory rhyme for months. How many are a dozen? A score? A gross? A great gross? Give the stationers' table.

What is meant by the reduction of a denominate number to one of a lower denomination? Illustrate. What is meant by the reduction of a denominate number to one of a higher denomination? Illustrate. What names are given to these two reductions? Describe the method used in each reduction. Illustrate each.

Describe the process of adding two or more compound denominate numbers; of subtracting one compound denominate number from another. How do these differ from ordinary addition and subtraction? How is the difference between two dates found?

By what kind of a number, only, can a compound denominate number be multiplied or divided? Solve an example in multiplication of compound denominate numbers and explain the steps in the process. Solve an example in division of compound denominate numbers and explain the steps in the process.

PRACTICAL MEASUREMENTS.—Define right angle; two lines perpendicular to each other; parallel lines; parallelogram; rectangle; base of a figure; altitude of a figure; diagonal of a parallelogram; area. Illustrate each. Give the rule for finding the area of a rectangular surface. How is this rule obtained? Illustrate by a drawing. Give a rule for finding the area of a parallelogram. Show how this rule is obtained. Give a rule for finding the area of a triangle. Show how this is obtained. Indicate the base and altitude of the triangle.

How is the cost of plastering and painting estimated? What allowance should be made for doors and windows? In what widths is carpet commonly sold? How is the amount of carpet required to cover the floor of a room determined? What is the length of a roll of paper? What is the width, usually? How is the amount of paper required to cover the walls and ceiling of a room determined?

How is the volume of a rectangular solid computed? How is wood bought and sold? Define a cord. How many cubic feet are there in a cord? In a cord foot? In a perch of stone?

How is lumber 1 inch thick measured? Illustrate. How is lumber more than 1 inch thick measured? What is

meant by a thousand feet of lumber? How is the capacity of rectangular bins, boxes, etc., measured? Of rectangular tanks, cisterns, etc.?

PERCENTAGE. — Define and illustrate the term per cent. What is the sign of per cent? How is per cent usually expressed? In what other ways may it be expressed? Give several simple fractions and their equivalents expressed as per cent. Define and illustrate base, rate, and percentage. What letters are commonly used to stand for base, rate, and percentage?

What must be known, or given, to find the percentage? The base? The rate? What are the three cases in percentage? Give a rule for finding the percentage when the base and rate are given, and illustrate by solving an example. Express this rule in terms of B , R , and P . Give a rule for finding the rate when the percentage and base are given, and illustrate by solving an example. Give the rule in terms of B , R , and P . Give a rule for finding the base when the percentage and rate are given, and illustrate by solving an example. Give the rule in terms of B , R , and P .

On what sum are gains and losses reckoned? What is the nature of the business of an agent, or commission merchant, or broker? Illustrate. What is meant by commission or brokerage? Illustrate. Define net proceeds. Illustrate. In solving problems in commission, upon what is the agent's commission reckoned?

Define price list; commercial discount; net amount. Illustrate these terms. When several discounts are allowed, upon what are they computed? Illustrate.

Define insurance. What two classes of insurance are there? What is property insurance? Illustrate. What are the principal varieties of property insurance? What is personal insurance? What are the principal kinds of personal insurance? What is a policy? A life policy? An

accident policy? An endowment policy? What is the premium?

INTEREST.—Define interest; principal; amount. Illustrate. In computing interest how many days are regarded as a year? A month? How is the interest found from the principal, rate, and time? How is the amount found? At 6%, what is the interest of \$1 for 1 year? For any number of months? For any number of days? How is interest computed by the six per cent method, if the rate is 7%? If the rate is 5%?

What is a promissory note? Who is the maker, or drawer? The payee? The holder? Illustrate. What is meant by the face of the note? Where does the maker of a note write his name? Where does he write the amount he promises to pay? Where does he write the payee's name? What specifications should be given in a note?

If no rate of interest is specified, what rate is understood? If the words "with interest" are omitted, when will the note begin to draw interest?

How can Mr. Jones guarantee the payment of a note given by Mr. Brown to Mr. White? If Mr. Brown were unable to pay the note when due, who would be obliged to pay it? What is meant by the indorser of a note? What is a negotiable note? To whom must a negotiable note be made payable? What is a non-negotiable note? If the drawer of a note fails to pay it, through inability or fraud, what person or persons, if any, must pay it? Give two forms of indorsement. What is meant by "days of grace"? When does a note mature?

What is a bank? In what does a large part of the business of most banks consist? Define term of discount; bank discount. Where are notes discounted at a bank usually made payable? Define proceeds of a note. How is the term of discount usually reckoned?

ANSWERS.

Page 22.—2. 283. 3. 254. 4. 231. 5. 212. 6. 136. 7. 185.
8. 300. 9. 198. 10. 198. 11. 280. 12. 315. 13. 274. 14. 213.
15. 152. 16. 107. 17. 260. 18. 226. 19. 289. 20. 233. 21. 313.
22. 293. 23. 197. 24. 179. 25. 271.

Page 23.—26. 46 mi. 27. \$57. 28. 88 yr. 29. 100 articles.
30. 38 boys; 41 girls; 79 children. 31. 134 fishes. 32. 180 pounds.
33. 262 pages. 34. 168 acres. 35. 31 mi. 36. 44 examples. 37. 79¢.
38. 87 bu.

Page 24.—39. 50 chickens. 40. \$57. 41. 93¢. 42. \$1.32.
43. 105 cows. 44. \$1.83. 45. 112 things. 46. 126 marbles.
47. 97 leaves. 48. 121 loaves. 49. 262 qt. 50. 148 lb.

Page 25.—51. 88 animals. 52. \$25. 53. \$40. 54. 252. 55. 266
yd. 56. \$88, horse; \$176, together. 57. \$1.30. 58. 123 lines.
59. \$1821. 60. 1335 acres. 61. 3198 pupils in all; 1401, 4th school.

Page 26.—62. 593. 63. 470. 64. 541. 65. 467. 66. 461. 67. 365.
68. 542. 69. 5116. 70. 5048. 71. 3681. 72. 4129. 73. 5415.
74. 5311. 75. 71,425. 76. 67,053. 77. 64,794. 78. 48,551.
79. 23,437.

Page 28.—2. 532. 3. 723. 4. 351. 5. 413. 6. 214. 7. 441.
8. 633. 9. 201. 10. 515. 11. 543. 12. 410. 13. 662. 14. 126.
15. 456. 16. 634. 17. 613. 18. 245. 19. 724. 20. 654.

Page 29.—24. 17. 25. 17. 26. 37. 27. 23. 28. 41. 29. 38.
30. 16. 31. 23. 32. 54. 33. 24. 34. 34. 35. 48. 36. 3. 37. 68.
38. 55. 39. 37. 40. 614. 41. 414. 42. 222. 43. 27. 44. 513.
45. 223. 46. 110. 47. 208. 48. 129. 49. 126. 50. 317. 51. 428.
52. 781. 53. 292. 54. 221.

Page 30.—55. 166. 56. 323. 57. 468. 58. 227. 59. 414. 60. 601.
61. 209. 62. 153. 63. 625. 64. 434. 65. 657. 66. 609. 67. 641.
68. \$49. 69. \$721. 70. 169 children. 71. 281 eggs. 72. 758 birds.
73. 79 pints. 74. 115 horses. 75. 265 cows. 76. \$432. 77. 531 bu.
78. 635 apples. 79. 365 da. 80. 56 yr. 81. \$148. 82. \$188.

Page 31.—1. 534 gal. 2. 545 mi. 3. 3621 bricks. 4. Lost \$22.
5. 594. 6. \$195. 7. 221 mi. 8. Answers differ. 9. 836 yd.
10. \$5569, entire cost; \$2375 more. 11. Answers differ. 12. 112 ft.

Page 32.—13. \$289. 14. 451 apple trees; 715 trees. 15. 34 bu.
16. \$97.49. 17. \$2.87. 18. \$89. 19. \$2058; \$431.50. 20. \$1262.
21. 237 mi. 22. 613 mi. 23. 108 pupils. 24. \$556. 25. 58¢.

Page 38.—26. 375. 27. 225 pages. 28. \$1.00. 29. 430 pupils.
30. 207 bu. 31. \$3.25. 32. \$101.11. 33. \$351.60. 34. \$104.64.

35. \$90.64. 36. \$11.65. 37. \$34.99. 38. \$86.23. 39. \$44.73.
40. \$109.95. 41. \$77.91.

Page 39.—2. 230. 3. 222. 4. 196. 5. 558. 6. 602. 7. 392.
8. 207. 9. 464. 10. 387. 11. 510. 12. 957. 13. 1104. 14. 2905.
15. 2976. 16. 4230. 17. 1456. 18. 2754. 19. 1625. 20. 7776.
21. 1800. 22. 4896. 23. 6524. 24. 5709. 25. 10,044. 26. 12,980.
27. 14,805. 28. 11,826. 29. 34,122. 30. 30,672. 31. 21,116.
32. 73,512. 33. 27,538. 34. 27,340. 35. 6,346. 36. 47,124.
37. 23,844. 38. 26,400 ft. 39. 195 yd. 40. \$187.50. 41. \$66.00.
42. 376 mi. 43. \$426.25. 44. \$190.80.

Page 40.—45. \$630.00. 46. \$91.75. 47. \$58,600. 48. \$25,722.00.
49. \$71,667. 50. 50,592 papers. 51. \$698.80. 52. \$832.95. 53. 1000
tons. 54. 230,370 passengers. 55. \$269,465. 56. \$1921.35. 57. 83,208.
58. 22,648 mi.

Page 41.—59. 68,355 bunches. 60. 11,738; 17,607; 23,476;
29,345; 35,214; 41,083; 46,952; 52,821. 61. 14,592; 21,888; 29,184;
36,480; 43,776; 51,072; 58,368; 65,664. 62. 16,328; 24,492; 32,656;
40,820; 48,984; 57,148; 65,312; 73,476. 63. 7908; 11,862; 15,816;
19,770; 23,724; 27,678; 31,632; 35,586. 64. 7714; 11,571; 15,428;
19,285; 23,142; 26,999; 30,856; 34,713. 65. 11,358; 17,037; 22,716;
28,395; 34,074; 39,753; 45,432; 51,111. 66. 173,946; 260,919;
347,892; 434,865; 521,838; 608,811; 695,784; 782,757. 67. 47,094;
70,641; 94,188; 117,735; 141,282; 164,829; 188,376; 211,923.
68. 130,264; 195,396; 260,528; 325,660; 390,792; 455,924; 521,056;
586,188. 69. 37,144; 55,716; 74,288; 92,860; 111,432; 130,004;
148,576; 167,148. 70. 84,728; 127,092; 169,456; 211,820; 254,184;
296,548; 338,912; 381,276. 71. 76,906; 115,359; 153,812; 192,265;
230,718; 269,171; 307,624; 346,077. 72. 96,724; 145,086; 193,448;
241,810; 290,172; 338,534; 386,896; 435,258. 73. 65,146; 97,719;
130,292; 162,865; 195,438; 228,011; 260,584; 293,157. 74. 143,370;
215,055; 286,740; 358,425; 430,110; 501,795; 573,480; 645,165.

Page 42.—1. 450. 2. 670. 3. 380. 4. 460. 5. 760. 6. 32,400.
7. 41,600. 8. 72,800. 9. 34,200. 10. 51,800. 11. 5,463,000.
12. 3,784,000. 13. 51,690,000. 14. 42,370,000. 15. 819,300,000.

Page 43.—17. 900. 18. 1110. 19. 2720. 20. 5840. 21. 3360.
22. 3600. 23. 1500. 24. 2070. 25. 5880. 26. 3900. 27. 178,000.
28. 83,700. 29. 362,600. 30. 237,600. 31. 154,000. 32. 298,800.
33. 272,800. 34. 191,400. 35. 345,500. 36. 177,600. 37. 23,676,000.
38. 21,116,000. 39. 58,555,000. 40. 65,034,000. 41. 43,096,000.
42. 36,970,000. 43. 8,607,000. 44. 19,998,000. 45. 35,060,000.
46. 55,092,000.

Page 44.—51. 6408. 52. 22,572. 53. 55,998. 54. 11,742.
55. 12,555. 56. 40,260. 57. 51,471. 58. 33,432. 59. 12,354.
60. 69,768. 61. 70,308. 62. 22,240. 63. 59,616. 64. 45,936.
65. 16,016. 66. 907,420. 67. 2,256,404. 68. 2,237,340. 69. 3,255,112.
70. 1,640,536. 71. 3,642,690. 72. 1,062,152. 73. 1,330,329.
74. 6,780,025. 75. 6,938,840. 76. 2,883,686. 77. 8,189,100.
78. 5,836,005. 79. 4,965,954. 80. 3,468,255. 81. 4,512,816.
82. 2,749,835. 83. 5,996,046. 84. 4,323,284. 85. 2,673,054.
86. 4,846,122. 87. 2,353,176. 88. 27,381,900. 89. 29,277,186.

90. 60,355,476. 91. 302,422,875. 92. 468,083,374. 93. 186,173,400.
94. 307,907,152. 95. 447,832,464.

Page 45.—96. \$28,189.68. 97. \$14,795.60. 98. \$52,071.30.
99. \$71,125.30. 100. \$29,647.02. 101. \$2,164,284.20. 102. \$2,053,-
983.00. 103. \$2,884,098.15. 104. \$1,450,457.60. 105. \$2,774,732.64.
106. \$2808.65. 107. 195,360 ft. 108. \$92,022. 109. 441,066 bricks.
110. \$16,686.42. 111. \$516.46. 112. 14,208 hides. 113. \$4847.04.
114. 49,680 pencils. 115. 525,600 min. 116. \$3538.75. 117. 207,360
times. 118. 3,747,090.

Page 46.—119. 11,341,784 gal. 120. \$6,356,220. 121. \$444,475.
122. 160,965 bbl. 123. \$75,209.40.

Page 48.—1. \$220. 2. \$16,675. 3. \$950. 4. \$8024. 5. \$321.50.
6. \$184.81. 7. A, \$4918; B, \$4295.

Page 49.—8. \$243.25. 9. \$653.75. 10. \$332.75. 11. \$10,136.50.
12. \$1924.25. 13. \$11,340. 14. \$8280. 15. \$77 loss. 16. \$32,810.
17. \$773.70. 18. \$6840.

Page 50.—19. \$2528.75. 20. \$31,298.75. 21. \$12,799.60.
22. 1740 people. 23. \$13,833. 24. 10,720,000 seeds. 25. 252,018,000
fishes. 26. 147 mi. 27. 123 mi.

Page 55.—6. 1218. 7. 1366. 8. 594. 9. 496. 10. 1545. 11. 984.
12. 1078. 13. 876. 14. 454. 15. 166. 16. 125. 17. 211. 18. 554.
19. 1353. 20. 548.

Page 56.—21. 1645 $\frac{1}{2}$. 22. 913 $\frac{1}{2}$. 23. \$14.88 $\frac{1}{2}$. 24. \$9.08 $\frac{1}{2}$.
25. \$10,428. 26. \$14,653. 27. \$8374 $\frac{1}{2}$. 28. \$2296 $\frac{1}{2}$. 29. \$12,052.
30. \$4857 $\frac{1}{2}$. 31. \$9479 $\frac{1}{2}$. 32. \$10,149. 33. \$12,024. 34. \$2862 $\frac{1}{2}$.
35. 454. 36. 237. 37. 391. 38. 459. 39. 13,170 $\frac{1}{2}$. 40. 17,897.
41. 4390 $\frac{1}{2}$. 42. 450 coats. 43. 211 weeks. 44. \$753. 45. 156 hr.
46. 392 hr. 47. 689 people. 48. 3271 yd. 49. 6523 cloaks. 50. 586
fathoms. 51. 650; 487 $\frac{1}{2}$; 390; 325; 278 $\frac{1}{2}$; 243 $\frac{1}{2}$; 216 $\frac{1}{2}$. 52. 492; 369;
295 $\frac{1}{2}$; 246; 210 $\frac{1}{2}$; 184 $\frac{1}{2}$; 164. 53. 1453 $\frac{1}{2}$; 1090; 872; 726 $\frac{1}{2}$; 622 $\frac{1}{2}$;
545; 484 $\frac{1}{2}$. 54. 981; 735 $\frac{1}{2}$; 588 $\frac{1}{2}$; 490 $\frac{1}{2}$; 420 $\frac{1}{2}$; 367 $\frac{1}{2}$; 327. 55. 1872;
1404; 1123 $\frac{1}{2}$; 936; 802 $\frac{1}{2}$; 702; 624. 56. 2303; 1727 $\frac{1}{2}$; 1381 $\frac{1}{2}$; 1151 $\frac{1}{2}$;
987; 863 $\frac{1}{2}$; 767 $\frac{1}{2}$. 57. 872; 654; 523 $\frac{1}{2}$; 436; 373 $\frac{1}{2}$; 327; 290 $\frac{1}{2}$.
58. 628; 471; 376 $\frac{1}{2}$; 314; 269 $\frac{1}{2}$; 235 $\frac{1}{2}$; 209 $\frac{1}{2}$. 59. 5762; 4321 $\frac{1}{2}$;
3457 $\frac{1}{2}$; 2881; 2469 $\frac{1}{2}$; 2160 $\frac{1}{2}$; 1920 $\frac{1}{2}$. 60. 3871 $\frac{1}{2}$; 2903 $\frac{1}{2}$; 2323; 1935 $\frac{1}{2}$;
1659 $\frac{1}{2}$; 1451 $\frac{1}{2}$; 1290 $\frac{1}{2}$. 61. 14,951 $\frac{1}{2}$; 11,213 $\frac{1}{2}$; 8970 $\frac{1}{2}$; 7475 $\frac{1}{2}$; 6407 $\frac{1}{2}$;
5606 $\frac{1}{2}$; 4983 $\frac{1}{2}$. 62. 6269 $\frac{1}{2}$; 4702 $\frac{1}{2}$; 3761 $\frac{1}{2}$; 3134 $\frac{1}{2}$; 2687; 2351 $\frac{1}{2}$; 2089 $\frac{1}{2}$.
63. 92,407 $\frac{1}{2}$; 69,305 $\frac{1}{2}$; 55,444 $\frac{1}{2}$; 46,203 $\frac{1}{2}$; 39,603 $\frac{1}{2}$; 34,652 $\frac{1}{2}$; 30,802 $\frac{1}{2}$.
64. 182,779; 137,084 $\frac{1}{2}$; 109,667 $\frac{1}{2}$; 91,389 $\frac{1}{2}$; 78,333 $\frac{1}{2}$; 68,542 $\frac{1}{2}$; 60,926 $\frac{1}{2}$.
65. 129,882 $\frac{1}{2}$; 97,412; 77,929 $\frac{1}{2}$; 64,941 $\frac{1}{2}$; 55,664; 48,706; 43,294 $\frac{1}{2}$.
66. 87,316 $\frac{1}{2}$; 65,487 $\frac{1}{2}$; 52,390; 43,658 $\frac{1}{2}$; 37,421 $\frac{1}{2}$; 32,743 $\frac{1}{2}$; 29,105 $\frac{1}{2}$.

Page 58.—4. 372. 5. 486. 6. 387 $\frac{2}{5}$. 7. 426 $\frac{2}{5}$. 8. 384 $\frac{2}{5}$.
9. 573 $\frac{3}{10}$. 10. 425 $\frac{7}{10}$. 11. 298 $\frac{3}{10}$. 12. 342 $\frac{1}{10}$. 13. 513 $\frac{1}{10}$.
14. 52 $\frac{3}{10}$. 15. 32 $\frac{3}{10}$. 16. 97 $\frac{1}{10}$. 17. 54 $\frac{3}{10}$. 18. 69 $\frac{1}{10}$.
20. 341 $\frac{1}{10}$. 21. 243 $\frac{1}{10}$. 22. 81 $\frac{3}{10}$. 23. 49 $\frac{3}{10}$. 24. 329 $\frac{4}{10}$. 25. 95 $\frac{1}{10}$.
26. 58 $\frac{3}{10}$. 27. 68 $\frac{3}{10}$. 28. 166 $\frac{3}{10}$. 29. 39 $\frac{3}{10}$. 30. 96 $\frac{3}{10}$. 31. 135 $\frac{3}{10}$.

Page 60.—36. 382. 37. 213. 38. 66. 39. 48. 40. 55. 41. 73.

42. 456. 47 144. 44. 67. 45. 82. 46. 37. 47. 36. 48. 1829 $\frac{1}{2}$;
 1200 $\frac{1}{2}$; 893 $\frac{1}{2}$; 711 $\frac{1}{2}$; 591 $\frac{1}{2}$. 49. 3118 $\frac{1}{2}$; 2046 $\frac{1}{2}$; 1522 $\frac{1}{2}$; 1212 $\frac{1}{2}$;
 1007 $\frac{1}{2}$. 50. 4507 $\frac{1}{2}$; 2957 $\frac{1}{2}$; 2201 $\frac{1}{2}$; 1752 $\frac{1}{2}$; 1456 $\frac{1}{2}$. 51. 2175 $\frac{1}{2}$;
 1427 $\frac{1}{2}$; 1062 $\frac{1}{2}$; 845 $\frac{1}{2}$; 702 $\frac{1}{2}$. 52. 4034 $\frac{1}{2}$; 2647 $\frac{1}{2}$; 1970 $\frac{1}{2}$; 1569 $\frac{1}{2}$;
 1803 $\frac{1}{2}$. 53. 3054 $\frac{1}{2}$; 2004 $\frac{1}{2}$; 1491 $\frac{1}{2}$; 1187 $\frac{1}{2}$; 986 $\frac{1}{2}$. 54. 4755 $\frac{1}{2}$;
 3121 $\frac{1}{2}$; 2322 $\frac{1}{2}$; 1849 $\frac{1}{2}$; 1536 $\frac{1}{2}$. 55. 979 $\frac{1}{2}$; 642 $\frac{1}{2}$; 478 $\frac{1}{2}$; 380 $\frac{1}{2}$;
 316 $\frac{1}{2}$. 56. 3386 $\frac{1}{2}$; 2222 $\frac{1}{2}$; 1653 $\frac{1}{2}$; 1316 $\frac{1}{2}$; 1094 $\frac{1}{2}$. 57. 1928 $\frac{1}{2}$;
 1265 $\frac{1}{2}$; 941 $\frac{1}{2}$; 750 $\frac{1}{2}$; 623 $\frac{1}{2}$. 58. 1704 $\frac{1}{2}$; 1118 $\frac{1}{2}$; 832 $\frac{1}{2}$; 662 $\frac{1}{2}$;
 550 $\frac{1}{2}$. 59. 646 $\frac{1}{2}$; 424 $\frac{1}{2}$; 315 $\frac{1}{2}$; 251 $\frac{1}{2}$; 208 $\frac{1}{2}$. 60. 4237 $\frac{1}{2}$; 2781;
 2069 $\frac{1}{2}$; 1648; 1369 $\frac{1}{2}$. 61. 2229 $\frac{1}{2}$; 1463 $\frac{1}{2}$; 1088 $\frac{1}{2}$; 867 $\frac{1}{2}$; 720 $\frac{1}{2}$.
 62. 4566 $\frac{1}{2}$; 2996 $\frac{1}{2}$; 2230 $\frac{1}{2}$; 1775 $\frac{1}{2}$; 1475 $\frac{1}{2}$. 63. 33,515 $\frac{1}{2}$; 21,994 $\frac{1}{2}$;
 16,367 $\frac{1}{2}$; 13,033 $\frac{1}{2}$; 10,828 $\frac{1}{2}$. 64. 23,453 $\frac{1}{2}$; 15,391 $\frac{1}{2}$; 11,454 $\frac{1}{2}$;
 9120 $\frac{1}{2}$; 7577 $\frac{1}{2}$. 65. 31,859 $\frac{1}{2}$; 20,907 $\frac{1}{2}$; 15,559 $\frac{1}{2}$; 12,389 $\frac{1}{2}$;
 10,293 $\frac{1}{2}$. 66. 35,697 $\frac{1}{2}$; 23,426 $\frac{1}{2}$; 17,433 $\frac{1}{2}$; 13,882 $\frac{1}{2}$; 11,532 $\frac{1}{2}$.
 67. 39,920 $\frac{1}{2}$; 26,197 $\frac{1}{2}$; 19,495 $\frac{1}{2}$; 15,524 $\frac{1}{2}$; 12,897 $\frac{1}{2}$. 68. 1024 $\frac{1}{2}$;
 1353 $\frac{1}{2}$; 1995; 824 $\frac{1}{2}$; 1083. 69. 970 $\frac{1}{2}$; 1282 $\frac{1}{2}$; 1890 $\frac{1}{2}$; 780 $\frac{1}{2}$;
 1026 $\frac{1}{2}$. 70. 2665 $\frac{1}{2}$; 3522 $\frac{1}{2}$; 5191 $\frac{1}{2}$; 2144 $\frac{1}{2}$; 2818 $\frac{1}{2}$. 71. 1302 $\frac{1}{2}$;
 1721 $\frac{1}{2}$; 2537 $\frac{1}{2}$; 1048; 1377 $\frac{1}{2}$. 72. 806 $\frac{1}{2}$; 1065 $\frac{1}{2}$; 1569 $\frac{1}{2}$; 648 $\frac{1}{2}$;
 852 $\frac{1}{2}$. 73. 1374 $\frac{1}{2}$; 1815 $\frac{1}{2}$; 2676 $\frac{1}{2}$; 1105 $\frac{1}{2}$; 1452 $\frac{1}{2}$. 74. 1960 $\frac{1}{2}$;
 2591 $\frac{1}{2}$; 3818 $\frac{1}{2}$; 1577 $\frac{1}{2}$; 2073 $\frac{1}{2}$. 75. 2596 $\frac{1}{2}$; 3430 $\frac{1}{2}$; 5055 $\frac{1}{2}$;
 2088 $\frac{1}{2}$; 2744 $\frac{1}{2}$. 76. 518 $\frac{1}{2}$; 685 $\frac{1}{2}$; 1010 $\frac{1}{2}$; 417 $\frac{1}{2}$; 548 $\frac{1}{2}$. 77. 930 $\frac{1}{2}$;
 1230; 1812 $\frac{1}{2}$; 748 $\frac{1}{2}$; 984. 78. 1810 $\frac{1}{2}$; 2392 $\frac{1}{2}$; 3525 $\frac{1}{2}$; 1456 $\frac{1}{2}$;
 1913 $\frac{1}{2}$. 79. 2317 $\frac{1}{2}$; 3062 $\frac{1}{2}$; 4512 $\frac{1}{2}$; 1864 $\frac{1}{2}$; 2449 $\frac{1}{2}$. 80. 12,698 $\frac{1}{2}$;
 16,779 $\frac{1}{2}$; 24,728; 10,213 $\frac{1}{2}$; 13,423 $\frac{1}{2}$. 81. 7289 $\frac{1}{2}$; 9633; 14,196;
 5863 $\frac{1}{2}$; 7706 $\frac{1}{2}$. 82. 3336 $\frac{1}{2}$; 4409 $\frac{1}{2}$; 6497 $\frac{1}{2}$; 2683 $\frac{1}{2}$; 3527 $\frac{1}{2}$.
 83. 21,019 $\frac{1}{2}$; 27,776 $\frac{1}{2}$; 40,933 $\frac{1}{2}$; 16,907 $\frac{1}{2}$; 22,221 $\frac{1}{2}$. 84. 541.
 85. 723. 86. 2767 $\frac{1}{2}$. 87. 725 $\frac{1}{2}$.

Page 63.—1. \$61. 2. 23 yd. 3. 248 weeks.

Page 64.—4. \$130; \$650. 5. 144 doz. 6. \$53.35. 7. 128 trees. 8. 868 times. 9. 301 acres. 10. 2240 lb. 11. 150 da.; 2 $\frac{1}{2}$ da. 12. 72 hr.; 3 da. 13. 1569. 14. 2311, multiplicand. 15. 467. 16. 92 $\frac{1}{2}$ lb.; sister, 83 $\frac{1}{2}$ lb. 17. \$51,614. 18. \$3080. 19. 97 schooners. 20. 23 $\frac{1}{2}$ gal. 21. 259 families.

Page 65.—22. 1000. 23. 43 cars. 24. 234 da. 25. 5130 shares. 26. 5 $\frac{3}{4}$ mi. 27. 6272. 28. \$224. 29. 352 times. 30. 27 posts; 104 rails. 31. 488 acres. 32. A, \$208; B, \$224. 33. 3535; 3207. 34. 779 gal.

Page 66.—35. \$4.64. 36. Elsie, \$2.65; Lottie, \$3.05. 37. 14 da. 38. \$584. 39. 629 mi. 40. \$158. 41. \$1816. 42. \$1149.12. 43. 472 bu. 44. 12 horses; 41 sheep. 45. 25 times. 46. 16 da. 47. Gained \$161. 48. 50 gross.

Page 67.—49. \$75.60. 50. 5 yr. 51. \$98.79. 52. 10 $\frac{1}{2}$ yr. 53. Gained \$40. 54. 938 mi. 55. 98 mi. 56. A, \$46; B, \$53; C, \$99. 57. Howard, 35 fishes; Jamie, 23 fishes; Roy, 19 fishes. 58. \$198. 59. \$1410. 60. \$396.

Page 68.—61. \$4725. 62. B, 442 acres; C, 1284 acres; D, 1511 acres. 63. 76 lb. 64. 8 yr. 65. \$55. 66. 406. 67. 22,962. 68. 42,009.

Page 69.—1. 55. 2. 39. 3. 10. 4. 2. 5. 8. 6. 5. 7. 21. 8. 5. 9. 36. 10. 4. 11. 16. 12. 36. 13. 58. 14. 10. 15. 117. 16. 40. 17. 112. 18. 56. 19. 109. 20. 60. 21. 157. 22. 7.

Page 73.—2. 2, 2, 2, 3, 7. 3. 2, 2, 2, 2, 2, 7. 4. 2, 2, 2, 2, 3, 3. 5. 2, 3, 3, 11. 6. 2, 2, 2, 3, 3, 3. 7. 2, 2, 11, 11. 8. 2, 2, 2, 2, 2, 2, 3, 3. 9. 2, 2, 2, 2, 3, 3, 3. 10. 2, 2, 2, 2, 2, 3, 7. 11. 2, 2, 3, 3, 11. 12. 5, 5, 5, 5. 13. 2, 2, 2, 2, 3, 19. 14. 2, 2, 2, 2, 2, 2, 13. 15. 2, 3, 3, 5, 11. 16. 2, 2, 2, 2, 3, 3, 7. 17. 3, 5, 29. 18. 2, 3, 37. 19. 2, 2, 2, 2, 2, 3, 7. 20. 5, 5, 31. 21. 17, 17. 22. 2, 2, 2, 2, 2, 2, 2, 2, 5. 23. 3, 5, 5, 5, 7. 24. 2, 2, 3, 3, 89. 25. 3, 5, 7, 11. 26. 2, 2, 5, 5, 5, 5. 27. 3, 3, 3, 5, 11. 28. 2, 2, 2, 2, 3, 3, 3, 3. 29. 2, 2, 2, 2, 2, 3, 3, 7. 30. 5, 5, 5, 23. 31. 2, 5, 17, 17.

Page 74.—3. $2\frac{1}{2}$. 4. $1\frac{1}{2}$.

Page 75.—5. 4. 6. 16. 7. 3. 8. $27\frac{1}{2}$. 9. 40. 10. $18\frac{1}{2}$. 11. 45. 12. 6. 13. 4. 14. 60. 15. 40. 16. $3\frac{1}{8}$. 17. $16\frac{1}{2}$. 18. $6\frac{1}{2}$. 19. 8. 20. $2\frac{1}{2}$. 21. $1\frac{1}{2}$. 22. $3\frac{1}{2}$. 23. $6\frac{1}{2}$. 24. $\frac{1}{2}$. 25. 1. 26. $15\frac{1}{2}$ yd. 27. \$860. 28. \$3.12 $\frac{1}{2}$. 29. \$3.12.

Page 76.—30. 48¢. 31. $1\frac{1}{2}$ ¢. 32. \$48. 33. $53\frac{1}{2}$ yd. 34. 7. 35. $30\frac{1}{2}$. 36. $2\frac{1}{2}$. 37. $161\frac{1}{2}$. 38. $1\frac{1}{2}$. 39. $4\frac{1}{2}$. 40. $13\frac{1}{2}$.

Page 83.—2. $\frac{1}{10}$. 3. $\frac{1}{4}$. 4. $\frac{5}{8}$. 5. $\frac{10}{16}$. 6. $\frac{1}{2}$. 7. $\frac{3}{4}$. 8. $\frac{1}{2}$. 9. $\frac{2}{3}$. 10. $\frac{2}{3}$. 11. $\frac{3}{4}$. 12. $\frac{5}{8}$. 13. $\frac{2}{3}$. 14. $\frac{3}{4}$. 15. $\frac{2}{3}$. 16. $\frac{1}{2}$. 17. $\frac{1}{2}$. 18. $\frac{1}{2}$. 19. $\frac{2}{3}$. 20. $\frac{2}{3}$. 21. $\frac{2}{3}$.

Page 84.—2. $17\frac{1}{2}$. 3. $42\frac{1}{2}$. 4. $49\frac{1}{2}$. 5. $46\frac{1}{2}$. 6. $52\frac{1}{2}$. 7. $22\frac{1}{2}$. 8. $16\frac{1}{2}$. 9. $12\frac{1}{2}$. 10. $22\frac{1}{2}$. 11. $20\frac{1}{2}$. 12. $14\frac{1}{2}$. 13. $19\frac{1}{2}$. 14. $21\frac{1}{2}$. 15. $3\frac{1}{2}$. 16. $10\frac{1}{2}$. 17. $15\frac{1}{2}$. 18. $24\frac{1}{2}$. 19. $49\frac{1}{2}$. 20. $56\frac{1}{2}$. 21. $93\frac{1}{2}$.

Page 86.—2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $\frac{1}{2}$. 6. $\frac{1}{2}$. 7. $\frac{1}{2}$. 8. $\frac{1}{2}$. 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $\frac{1}{2}$. 16. $\frac{1}{2}$. 17. $\frac{1}{2}$. 18. $\frac{1}{2}$. 19. $\frac{1}{2}$. 20. $\frac{1}{2}$. 21. $\frac{1}{2}$. 22. $\frac{1}{2}$. 23. $\frac{1}{2}$. 24. $\frac{1}{2}$. 25. $\frac{1}{2}$.

Page 87.—2. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 3. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 4. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 5. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 6. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 7. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 8. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 9. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 10. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 11. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 12. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 13. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$.

Page 88.—15. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 16. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 17. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 18. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 19. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 20. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 21. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 22. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 23. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 24. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 25. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 26. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 27. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 28. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. 29. $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$.

Page 89.—2. $1\frac{1}{2}$. 3. $1\frac{1}{2}$. 4. $1\frac{1}{2}$. 5. $1\frac{1}{2}$. 6. $1\frac{1}{2}$. 7. $\frac{3}{4}$. 8. $1\frac{1}{2}$. 9. $1\frac{1}{2}$. 10. $13\frac{1}{2}$. 11. $10\frac{1}{2}$. 12. $16\frac{1}{2}$. 13. $16\frac{1}{2}$. 14. $12\frac{1}{2}$. 15. $23\frac{1}{2}$. 16. $21\frac{1}{2}$. 17. $17\frac{1}{2}$. 18. \$85. 19. $28\frac{1}{2}$ bu. 20. $698\frac{1}{2}$ lb. 21. $125\frac{1}{2}$ ft. 22. $8\frac{1}{2}$ o'clock.

Page 92.—3. $\frac{1}{2}$. 4. $\frac{1}{3}$. 5. $\frac{1}{4}$. 6. $\frac{1}{5}$. 7. $\frac{1}{6}$. 8. $\frac{1}{7}$. 9. $\frac{1}{8}$.

Page 93.—10. $\frac{1}{9}$. 11. $\frac{1}{10}$. 12. $\frac{1}{11}$. 13. $6\frac{1}{11}$. 14. $12\frac{1}{11}$.
15. $4\frac{1}{11}$. 16. $18\frac{1}{11}$. 17. $4\frac{1}{35}$. 18. $5\frac{1}{11}$. 19. $5\frac{1}{11}$. 20. $5\frac{1}{11}$. 21. $3\frac{1}{11}$.
22. $10\frac{1}{11}$. 23. $23\frac{1}{11}$. 24. $71\frac{1}{11}$ lb. 25. $83\frac{1}{11}$ acres. 26. $6\frac{1}{11}$ ft.
27. $30\frac{1}{11}$ lb. 28. $\$30\frac{1}{11}$. 29. $12\frac{1}{11}$. 30. $2\frac{1}{11}$. 31. $34\frac{1}{11}$ yd. 32. $\frac{1}{11}$.
Page 94.—33. $\frac{1}{11}$. 34. $2\frac{1}{11}$. 35. $\frac{1}{11}$. 36. $\frac{1}{11}$. 37. $\frac{1}{11}$. 38. $\frac{1}{11}$.
39. $\frac{1}{11}$. 40. $9\frac{1}{11}$. 41. $4\frac{1}{11}$. 42. $4\frac{1}{11}$. 43. $7\frac{1}{11}$. 44. $5\frac{1}{11}$.
45. $4\frac{1}{11}$. 46. $10\frac{1}{11}$.

Page 95.—2. $\frac{1}{11}$. 3. $\frac{1}{11}$. 4. $2\frac{1}{11}$. 5. $3\frac{1}{11}$. 6. $8\frac{1}{11}$. 7. 10.
8. $10\frac{1}{11}$. 9. $12\frac{1}{11}$. 10. $10\frac{1}{11}$. 11. $12\frac{1}{11}$. 12. $20\frac{1}{11}$. 13. $78\frac{1}{11}$. 14. $158\frac{1}{11}$.
15. $259\frac{1}{11}$. 16. $130\frac{1}{11}$.

Page 96.—17. $71\frac{1}{11}$ mi. 18. $247\frac{1}{11}$ mi. 19. $\$13.33\frac{1}{11}$. 20. $\$100\frac{1}{11}$.
21. 165 ft. 22. $16\frac{1}{11}$ cords. 23. 175 hr. 24. $\$67\frac{1}{11}$.

Page 97.—2. $6\frac{1}{11}$. 3. 15. 4. $24\frac{1}{11}$. 5. 21. 6. $12\frac{1}{11}$. 7. $20\frac{1}{11}$.
8. $47\frac{1}{11}$. 9. $19\frac{1}{11}$. 10. $17\frac{1}{11}$. 11. $17\frac{1}{11}$. 12. $38\frac{1}{11}$. 13. $19\frac{1}{11}$. 14. $22\frac{1}{11}$.
15. 85. 16. $225\frac{1}{11}$. 17. $241\frac{1}{11}$. 18. $21\frac{1}{11}$. 19. 528.

Page 98.—20. $45\frac{1}{11}$. 21. 100 acres. 22. $\$249\frac{1}{11}$. 23. $\$5.61$.
24. $452\frac{1}{11}$ mi. 25. $\$5.03\frac{1}{11}$. 26. $\$148\frac{1}{11}$.

Page 100.—2. $\frac{1}{11}$. 3. $\frac{1}{11}$. 4. $\frac{1}{11}$. 5. $\frac{1}{11}$. 6. $\frac{1}{11}$. 7. $\frac{1}{11}$. 8. $\frac{1}{11}$.
9. $\frac{1}{11}$. 10. $\frac{1}{11}$. 11. $\frac{1}{11}$. 12. $\frac{1}{11}$. 13. $\frac{1}{11}$. 14. 3. 15. $6\frac{1}{11}$.
16. $5\frac{1}{11}$. 17. $6\frac{1}{11}$. 18. $\frac{1}{11}$. 19. $1\frac{1}{11}$. 20. $4\frac{1}{11}$. 21. $26\frac{1}{11}$. 22. $\frac{1}{11}$.
23. $\frac{1}{11}$. 24. $\frac{1}{11}$. 25. $\frac{1}{11}$. 26. $14\frac{1}{11}$ ft.; $57\frac{1}{11}$ ft. 27. $\$12\frac{1}{11}$.
28. $\$58\frac{1}{11}$. 29. $168\frac{1}{11}$ acres. 30. $\$145\frac{1}{11}$. 31. $\$1.81\frac{1}{11}$. 32. Lining
 $\$2\frac{1}{11}$; Dress $\$22\frac{1}{11}$.

Page 103.—3. $\frac{1}{11}$. 4. $\frac{1}{11}$. 5. $\frac{1}{11}$. 6. $\frac{1}{11}$. 7. $\frac{1}{11}$. 8. $\frac{1}{11}$.
9. $\frac{1}{11}$. 10. $\frac{1}{11}$. 11. $\frac{1}{11}$. 12. $\frac{1}{11}$. 13. $\frac{1}{11}$. 14. $\frac{1}{11}$. 15. $\frac{1}{11}$.
16. $\frac{1}{11}$. 17. $\frac{1}{11}$. 18. $\frac{1}{11}$. 19. $\frac{1}{11}$. 20. $\frac{1}{11}$. 21. $\frac{1}{11}$. 22. $\frac{1}{11}$. 23. $\frac{1}{11}$.
24. $\frac{1}{11}$. 25. $\frac{1}{11}$. 26. $\frac{1}{11}$ da. 27. $\frac{1}{11}$ per lb. 28. $\$791\frac{1}{11}$.
29. $\frac{1}{11}$. 30. $\frac{1}{11}$ da.

Page 104.—31. $23\frac{1}{11}$ mi. per hr. 32. $12\frac{1}{11}$ sq. rd.

Page 105.—2. 30. 3. $36\frac{1}{11}$. 4. $42\frac{1}{11}$. 5. $43\frac{1}{11}$. 6. 54. 7. $62\frac{1}{11}$.
8. $42\frac{1}{11}$. 9. $29\frac{1}{11}$. 10. $79\frac{1}{11}$. 11. $62\frac{1}{11}$. 12. $72\frac{1}{11}$. 13. $76\frac{1}{11}$. 14. $47\frac{1}{11}$.
15. $36\frac{1}{11}$. 16. $45\frac{1}{11}$. 17. $129\frac{1}{11}$. 18. $1393\frac{1}{11}$. 19. $2575\frac{1}{11}$. 20. $899\frac{1}{11}$.
21. $1486\frac{1}{11}$. 22. $6\frac{1}{11}$. 23. $9\frac{1}{11}$. 24. $4\frac{1}{11}$. 25. $17\frac{1}{11}$. 26. $4\frac{1}{11}$.
27. $7\frac{1}{11}$. 28. $4\frac{1}{11}$. 29. $12\frac{1}{11}$. 30. $12\frac{1}{11}$. 31. $7\frac{1}{11}$. 32. $12\frac{1}{11}$. 33. $9\frac{1}{11}$.
34. $20\frac{1}{11}$. 35. $5\frac{1}{11}$. 36. $2\frac{1}{11}$. 37. $5\frac{1}{11}$. 38. $31\frac{1}{11}$ yd. 39. $23\frac{1}{11}$ rd.
40. $21\frac{1}{11}$ bbl. 41. $36\frac{1}{11}$ bbl. 42. 3 cases.

Page 107.—3. $1\frac{1}{11}$. 4. $1\frac{1}{11}$. 5. $2\frac{1}{11}$. 6. $3\frac{1}{11}$. 7. $1\frac{1}{11}$.
8. $1\frac{1}{11}$. 9. $10\frac{1}{11}$. 10. $2\frac{1}{11}$. 11. $3\frac{1}{11}$. 12. $1\frac{1}{11}$. 13. $6\frac{1}{11}$.
14. $1\frac{1}{11}$. 15. $5\frac{1}{11}$. 16. $5\frac{1}{11}$. 17. $40\frac{1}{11}$. 18. $30\frac{1}{11}$. 19. $4\frac{1}{11}$.
20. $5\frac{1}{11}$.

Page 108.—21. $12\frac{1}{11}$ times. 22. $7\frac{1}{11}$ bu. 23. $7\frac{1}{11}$ bbl.
24. $4\frac{1}{11}$ baskets. 25. 8 bbl. 26. $7\frac{1}{11}$ lb. 27. 10 books. 28. $7\frac{1}{11}$ doz.
29. 6 plows. 30. $5\frac{1}{11}$ acres. 31. $\$4056\frac{1}{11}$. 32. $5\frac{1}{11}$ bu. 33. $21\frac{1}{11}$ bu.

Page 109.—35. $\frac{22}{215}$. 36. $\frac{16}{235}$. 37. $\frac{1}{148}$. 38. $\frac{175}{8096}$. 39. $\frac{5}{1574}$.
40. $\frac{7}{32}$. 41. $\frac{4}{15}$. 42. $\frac{3}{345}$. 43. $\frac{1}{35}$. 44. $\frac{1}{40}$. 45. $\frac{1}{153}$. 46. $\frac{11}{15}$.
2. $2\frac{2}{3}$. 3. $1\frac{1}{15}$. 4. $1\frac{1}{3}$. 5. $1\frac{1}{2}$. 6. $6\frac{1}{2}$. 7. $11\frac{1}{2}$. 8. $2\frac{1}{2}$.

9. $1\frac{1}{2}$. 10. $1\frac{1}{8}$. 11. $1\frac{2}{75}$. 12. $\frac{3}{23}$. 13. $7\frac{1}{2}$.
Page 111.—2. $\frac{6}{17}$. 3. $\frac{1}{25}$. 4. $\frac{1}{32}$. 5. $\frac{1}{35}$. 6. $\frac{2}{88}$. 7. $\frac{1}{10}$.
8. $\frac{7}{200}$. 9. $\frac{1}{344}$. 10. $\frac{1}{88}$. 11. $\frac{11}{288}$. 12. $\frac{1}{17}$. 13. $\frac{1}{8}$. 14. $\frac{1}{12}$.
15. $\frac{8}{108}$. 16. $\frac{2}{9}$. 17. $\frac{1}{100}$. 18. $\frac{1}{21}$. 19. $\frac{37}{125}$. 20. $\frac{1}{144}$. 21. \$1.80.

Page 113.—2. 64. 3. 100. 4. 170. 5. 100. 6. 91. 7. 90.
8. 325. 9. 400. 10. 600. 11. 850. 12. 380. 13. 510. 14. 5.
15. $\frac{9}{20}$. 16. $1\frac{1}{15}$. 17. $1\frac{1}{2}$. 18. $1\frac{5}{7}$. 19. $\frac{5}{10}$.

Page 114.—20. \$120. 21. 100 ft. 22. 425 mi. 23. 150 ft.
24. 192 plants. 25. 55 mi.

Page 116.—1. $4\frac{1}{336}$. 2. $6\frac{12}{500}$. 3. $16\frac{227}{100}$. 4. $78\frac{141}{10}$. 5. $41\frac{11}{10}$.
6. $42\frac{1}{2}$. 7. 765. 8. $34\frac{11}{16}$.

Page 117.—9. \$84 $\frac{1}{2}$. 10. 40 bu. 11. \$25 $\frac{5}{7}$. 12. 211 $\frac{1}{2}$ bu.
13. \$4083 $\frac{1}{2}$. 14. \$6. 15. $\frac{1}{12}$. 16. 72 yr. 17. A, $\frac{1}{2}$ of work;
B, $\frac{1}{3}$ of work; both, $\frac{1}{6}$ of work. 18. 42 da.; $\frac{1}{3}$ of 42 da. = 3 $\frac{2}{3}$ da.

Page 118.—19. \$150,000. 20. $1\frac{1}{2}$ da. 21. $4\frac{1}{2}$ da. 22. 24 da.
23. \$620; \$465. 24. $2\frac{1}{2}$ da. 25. \$90,000. 26. $14\frac{1}{2}$. 27. $2\frac{1}{2}$.
28. 1. 29. $8\frac{5}{9}$. 30. \$312.50. 31. \$9000. 32. \$26,666 $\frac{2}{3}$.
33. 719 girls; 320 boys.

Page 119.—34. \$25,771 $\frac{2}{3}$. 35. $1\frac{4787}{2885}$. 36. $20\frac{25}{144}$ mi.
37. $3\frac{1}{2}$ in.; $19\frac{3}{4}$ in. 38. \$610. 39. \$250,000. 40. $\frac{389}{1379}$.
41. Increased $\frac{1}{2}$. 42. 600 lb. 43. \$ $\frac{3}{8}$ per lb. 44. \$34,600.
45. 37 $\frac{1}{2}$ da.

Page 124.—2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{7}{20}$. 5. $\frac{1}{2}$. 6. $\frac{1}{2}$. 7. $\frac{7}{8}$. 8. $\frac{1}{2}$.
9. $\frac{1}{2}$. 10. $\frac{9}{200}$. 11. $\frac{1}{20}$. 12. $\frac{7}{100}$. 13. $\frac{1}{20}$. 14. $\frac{1}{100}$. 15. $\frac{1}{100}$.
16. $\frac{4}{100}$. 17. $\frac{7}{80}$.

Page 125.—2. .3. 3. .625. 4. .1875. 5. .25. 6. .106+.
7. .15. 8. .025. 9. .0606+. 10. .316+. 11. .7105263+.
12. .45+. 13. .108+. 14. .428571+. 15. .36585+. 16. .0064.
17. 3.5. 18. 1.857142+. 19. 8.66+. 20. 3.166+. 21. 6.35.
22. .57 $\frac{1}{2}$. 23. .15625. 24. .0375. 25. .006.

2. 84.331. 3. 68.454. 4. 8.301. 5. 53.875.
Page 126.—6. 33.6635. 7. 64.435. 8. 5.1195. 9. 91.7472.
10. 430.435. 11. 8.615. 12. 49.1964. 13. 35.83262. 14. 67.8399.
15. 80.81. 16. 1.2119. 17. .9962493. 18. 1.938. 19. .3706883.
20. 133 miles. 21. \$38,741.875.

Page 127.—2. 11.33. 3. 10.095. 4. 27.995. 5. 25.46. 6. 12.012.
7. 36.213. 8. 25.2963. 9. 5.4798. 10. 4.08. 11. 8.81. 12. 5.2905.
13. 4.1125. 14. .585. 15. .161739. 16. 26.6299. 17. 12.81965.
18. 76.5466. 19. 6.3066. 20. 4.75 bbl. 21. \$25.75. 22. \$2624.13.

Page 128.—2. .1875. 3. .816. 4. 38.76. 5. 1.545. 6. .04371.
7. .34125. 8. .3285. 9. 1.19075. 10. 3.971.

Page 129. — 11. .013394. 12. 4.02732. 13. 99.8959. 14. 1.8509.
 15. 17.92431. 16. .0160655. 17. .00163644. 18. .001252394.
 19. .0082622. 20. .595875. 21. 53.9852. 22. .051255. 23. 276.5.
 24. 496.75. 25. 279.5. 26. 893.5. 27. 687.35. 28. 376.8. 29. 683.5.
 30. 763.4. 31. 899.2; 8992; 89920; 899200. 32. 6830.25; 68302.5;
 683025. 33. .765625 Sum greater. 34. 23.7375. 35. 3.03125.

36. 513.60. 37. \$30.50. 38. 391.865 acres. 39. 400.2 mi.
Page 130. — 2. 17.2811. 3. 11759.6. 4. 98.765. 5. \$26.06.
 6. \$129.62. 7. \$40.598. 8. \$5.875. 9. \$7.35. 10. \$7.455.
 11. 98.0625. 12. 58.545. 13. 138.668. 14. 1.4405. 15. 9.54.
 16. 769. 17. \$655.60. 18. \$4408.375.

Page 132. — 2. 21.7. 3. 3.9487+. 4. 3420. 5. 3.84. 6. 8710.
 7. 67.2. 8. .47. 9. 3.4. 10. .036. 11. 51.2. 12. 376. 13. .0036.
 14. .0131. 15. 1.461. 16. .167104+. 17. .213. 18. 5.5. 19. .036.
 20. 2500. 21. .0072. 22. .315. 23. 2,000,000. 24. 51.6. 25. 2.34.
 26. 100.2. 27. 64.2. 28. 6275. 29. .0496+. 30. 32,820.
 31. 1,000,000,000.

Page 133. — 33. 3.925. 34. 2.645. 35. .3695. 36. .04825.
 37. .03862. 38. .004285. 39. 26.7 yd. 40. 14.75 bbl. 41. 265.8 cords.

Page 134. — 2. 21.82. 3. \$1564.50. 4. \$10.20. 5. \$31.405.

Page 135. — 1. \$30. 2. \$785. 3. \$58.6625. 4. \$84.50.
 5. 7000 ft. 6. \$61.394. 7. \$4.875. 8. 8.368+ yr. 9. 1820 letters.
 10. 36 pairs. 11. \$64.25. 12. \$121.25.

Page 136. — 13. 13.5 hr. 14. \$17.07. 15. \$105.25. 16. 20¢.
 17. \$338. 18. \$1544.70. 19. \$.02 $\frac{2}{3}$ gain, \$59.00. 20. \$2360.
 21. 1155.95, tons sold; 159.05, tons melted. 22. 12 hr.

Page 147. — 2. 176 in. 3. 451 in. 4. 574 in. 5. 1007 in.
 6. 220 gi. 7. 436 gi. 8. 494 gi. 9. 607 gi. 10. 15,315 sec.
 11. 36,738 sec. 12. 66,035 sec. 13. 6728 oz. 14. 14,012 oz.
 15. 73,292 oz. 16. 139,769 oz. 17. 26,362 gr. 18. 48,248 gr.
 19. 31,569 gr. 20. 251 pt. 21. 367 pt. 22. 409 pt. 23. 930 in.
 24. 1082 in. 25. 1654 in. 26. 4482 sq. in. 27. 7732 sq. in.
 28. 13,012 $\frac{1}{2}$ sq. yd. 29. 39,480 $\frac{1}{2}$ sq. yd. 30. 110,677 cu. in.
 31. 197,706 cu. in. 32. 18,680 gr. 33. 27,220 gr. 34. 1698 sheets.
 35. 2122 sheets. 36. 20,400 gr. 37. 33,660 gr. 38. 555,900 sec.
 39. 575 pt. 40. 1758 in. 41. 1980 pt. 42. 28,438 sq. yd.
 43. 463,482 in. 44. 37 lb. 8 oz. 45. 3 yd. 1 ft. 3 $\frac{1}{2}$ in. 46. 5 da.
 10 hr. 40 min. 47. 1 ft. 3 $\frac{1}{2}$ in. 48. 2 pk. 3.2 qt. 49. 9 cwt.
 50. 10 cu. ft. 1382.4 cu. in. 51. 266 rd. 3 yd. 2 ft. 52. 68 sq. rd.
 17 sq. yd. 2 sq. ft. 82 $\frac{1}{2}$ sq. in. 53. 5 da. 2 hr. 38 min. 24 sec.
 54. 1 qt. 2 gi. 55. 1 ft. 5.1 in.

Page 148. — 3. 131 gal. 2 qt. 1 pt. 1 gi. 4. 398 gal. 1 pt. 5. 178
 gal. 3 qt. 1 pt. 6. 17 rd. 4 $\frac{1}{2}$ yd. 11 in. 7. 12 rd. 2 yd. 8 in.
 8. 8 rd. 1 ft. 9 in.

Page 149. — 9. 55 bu. 2 pk. 6 qt. 10. 73 bu. 5 qt. 1 pt. 11. 61 bu.
 2 pk. 5 qt. 1 pt. 12. 3 cwt. 22 lb. 12 oz. 13. 5 cwt. 7 lb. 15 oz.
 14. 2 T. 19 cwt. 83 lb. 15. 1 T. 17 cwt. 95 lb. 16. 25 wk. 3 da.
 13 hr. 17. 2 hr. 16 min. 8 sec. 18. 6 da. 20 hr. 54 min. 19. 3 sq. yd.
 55 sq. in. 20. 4 sq. yd. 5 sq. ft. 33 sq. in. 21. 3 R. 18 qr. 22 sheets.
 22. 4 R. 16 qr. 11 sheets. 23. 4 cu. ft. 952 cu. in. 24. 1 lb. 2 oz.

1 pwt. 10 gr. 25. 1 lb. 5 oz. 9 pwt. 10 gr. 26. 1 lb. 7 oz. 18 gr.
 28. $1\frac{1}{2}$ yd. 29. $\frac{7}{16}$ pk. 30. $1\frac{1}{2}$ bu. 31. $\frac{3}{4}$ gal. 32. $\frac{1}{4}$ bu.
 33. $\frac{3}{4}$ rd.

Page 150. — 2. 35 gal. 1 qt. 3. 38 bu. 2 pk. 4. 22 da. 13 hr.
 30 min. 5. 169 lb. 14 oz. 6. 35 lb. 8 oz. 17 pwt. 21 gr. 7. 70 yd.
 2 ft. 9 in. 8. 84 gal. 1 qt. 1 pt. 9. 79 A. 153 sq. rd. 10. 41 T. 31 lb.
 11. 23 yd. 1 ft. $10\frac{1}{2}$ in. 2. 5 bu. 1 pk. 4 qt.

Page 151. — 3. 6 gal. 2 qt. 1 pt. 3 gi. 4. 2 da. 22 hr. 10 min.
 5. 22 lb. 10 oz. 3 pwt. 6. 10 yd. 2 ft. 9 in. 7. 3 rd. $4\frac{1}{2}$ yd. 2 ft.
 8. 5 lb. 9 oz. 6 dr. 9. 10 A. 68 sq. rd. 10. 25 sq. yd. 1 sq. ft.
 95 sq. in. 11. 14 T. 17 cwt. 39 lb. 11 oz. 13. 51 yr. 5 mo. 5 da.
 14. 25 yr. 6 mo. 18 da. 15. 52 yr. 2 mo. 29 da. 16. 10 yr.
 10 mo. 17 da.

Page 152. — 2. 34 bu. 1 pk. 3 qt. 3. 34 gal. 1 pt. 4. 44 lb. 4 oz.
 2 pwt. 16 gr. 5. 54 lb. 1 oz. 2 sc. 10 gr. 6. 16 hr. 42 min. 55 sec.
 7. 18 T. 3 cwt. 91 lb. 8 oz. 8. 24 rd. 2 yd. 1 ft. 6 in. 9. 75 cu. yd.
 12 cu. ft. 1506 cu. in. 10. 41 sq. yd. 2 sq. ft. 81 sq. in. 11. 21 da.
 9 hr. 1 min. 40 sec. 12. 103 rd. 1 ft. 10 in. 13. 48 R. 18 qr.
 14. 44 bbl. 13 gal. 1 pt.

2. 3 gal. 2 qt. 1 pt. $1\frac{1}{2}$ gi. 3. 3 bu. 3 pk. 7 qt. $\frac{1}{2}$ pt. 4. 1 yd. 2 ft.
 $11\frac{1}{2}$ in. 5. 4 cwt. 73 lb. $11\frac{1}{2}$ oz. 6. 4 lb. 8 oz. 19 pwt. $10\frac{1}{2}$ gr.
 7. 2 lb. 9 oz. 7 dr. 2 sc. $2\frac{1}{2}$ gr. 8. 3 hr. 11 min. 46 sec. 9. 5 sq. yd.
 4 sq. ft. $11\frac{1}{2}$ sq. in. 10. 3 rd. 2 ft. $11\frac{1}{2}$ in.

Page 154. — 2. 391 sq. ft. 3. 192 sq. ft.; $21\frac{1}{2}$ sq. yd. 4. 48 sq. yd.
 5. 19,200 sq. rd.; 120 A. 6. \$17.578+. 7. 15 A., corn; $7\frac{1}{2}$ A.,
 potatoes.

Page 155. — 2. 120 sq. ft. 3. 7500 sq. rd. 4. 304 sq. ft.

Page 156. — 5. 96 A. 6. 8 ft.

2. 30 sq. in. 3. 60 sq. yd.

Page 157. — 4. 266 sq. ft. 5. 1080 sq. rd. 6. 7 A. 7. 25 ft.

1. \$27.14. 2. \$4.44. 3. \$95.70. 4. \$18.832+.

Page 158. — 2. \$60.00. 3. \$57.60. 4. 90 yd. 5. \$106.95.
 6. 43 yd.

Page 159. — 2. 13 rolls. 3. 6 rolls. 4. 10 double rolls and 1
 single roll, or 21 rolls. 5. 6 double rolls, 12 rolls; \$13.20.

Page 161. — 2. 420 cu. ft. 3. 792 cu. in. 4. 400 cu. yd.
 5. 2100 cu. ft. 6. 680 cu. ft. 7. 888 cu. ft. 8. 1260 cu. ft.
 9. $986\frac{1}{2}$ cu. ft. 10. 700 cu. ft. 11. 3 cd. 12. 8 cd. 13. 72 cd.
 14. \$396.

Page 162. — 15. \$78.222+. 16. \$36.363+.

1. 24 ft. 2. 15 ft. 3. $13\frac{1}{2}$ ft. 4. $23\frac{1}{2}$ ft. 5. $12\frac{1}{2}$ ft. 6. $26\frac{1}{2}$ ft.
 7. \$3.00. 8. 1440 ft. 9. \$25.92. 10. \$16.426+. 11. \$25.38.

Page 163. — 2. 33.75 bu. 3. 128.571+ bu. 4. 353.571+ bu.
 5. 60.267+ bu.

2. 523 $\frac{7}{16}$ gal. 3. 1009.87+ gal. 4. 822 $\frac{1}{2}$ gal. 5. 18.998+ bbl.

Page 166. — 2. \$8.43. 3. \$732.16. 4. \$333.27. 5. \$898.885.
 6. \$214.88. 7. \$792.54. 8. \$2.441. 9. \$9.376+.

Page 167. — 10. 91 sheep. 11. 195 cd. 12. \$10,880.
 13. \$207.36. 14. \$5540. 15. \$6310, son; \$3155, daughter.
 16. \$675.

Page 168.—2. 50%. 3. 30%. 4. $62\frac{1}{2}\%$. 5. 25%. 6. 24%.
7. 25%. 8. 50%. 9. 30%. 10. 75%. 11. 25%. 12. 60%.
13. 56%. 14. 55%.

Page 169.—15. 5% water; 95% vinegar. 16. 120 bu. wheat;
288 bu. oats; 72 bu. corn; 15% corn. 17. 5% killed; 16% wounded;
27% captured; 52% left.

Page 170.—2. 1500. 3. 1400. 4. 2080. 5. 1500. 6. 700.
7. 850. 8. 750. 9. 720. 10. 840. 11. 666. 12. 1600.
13. 300. 14. \$125. 15. \$200. 16. \$550. 17. 200 sheep.
18. \$300. 19. \$3600. 20. 500 bu. 21. \$8000.

Page 172.—1. \$2.56. 2. \$268.75. 3. \$1150. 4. 8%.
5. 14%. 6. 15%. 7. \$12,800. 8. 60¢ per bu. 9. 30¢ per lb.
10. $16\frac{2}{3}\%$. 11. \$36.00. 12. 15%. 13. \$1500.

Page 173.—14. \$49. 15. \$76,067. 16. 25%. 17. \$57.60.
18. 32%. 19. \$530. 20. \$4390. 21. 40¢. 22. 20%.
23. \$2898. 24. 12%. 25. 20%.

Page 174.—1. \$14.20. 2. \$32.25; \$2117.75.

Page 175.—3. \$7997.65. 4. \$23. 5. \$11. 6. 5%.
7. \$7.50. 8. 4%. 9. \$4280. 10. \$2337.40. 11. \$892.80.
12. \$1.31. 13. $4\frac{1}{2}\%$. 14. \$5625. 15. \$1734.50.

Page 176.—1. \$288. 2. \$513. 3. \$288. 4. \$527.
5. \$769.50. 6. \$532. 7. \$5.

Page 178.—1. \$39.60. 2. \$138. 3. \$309.60. 4. \$676.80.
5. \$375. 6. \$150. 7. \$78.75. 8. $\frac{1}{2}\%$. 9. \$29.50.
10. \$6000. 11. \$5000. 12. \$6. 13. \$1158.

Page 180.—2. \$25.48+. 3. \$85.224. 4. \$97.275.
5. \$61.376. 6. \$64.832. 7. \$174.69.

Page 181.—9. \$43.968. 10. \$66,855+. 11. \$93,907+.
12. \$171,971+. 13. \$180,742+. 14. \$40,853+. 15. \$125,147+.
17. \$47,281+; Amt. \$407,521+. 18. \$85,865+; Amt. \$501,345+.
19. \$22,098+; Amt. \$239,458+. 20. \$94,195+; Amt. \$517,545+.
21. \$130,729+; Amt. \$850,889+. 22. \$70,774+; Amt. \$539,134+.
23. \$32,262+; Amt. \$251,732+. 24. \$25,494+; Amt. \$350,834+.
25. \$91,508+; Amt. \$527,838+. 26. \$94.44+; Amt. \$391.89+.

Page 182.—2. \$49.946. 3. \$55.095+. 4. \$103,674+.
5. \$134,809+. 6. \$171,271. 7. \$168.112. 8. \$102,671+.
9. \$77.456.

Page 186.—3. \$3.75, discount; \$296.25, proceeds. 4. \$8.70,
discount; \$571.30, proceeds. 5. \$3,413+, discount; \$636.587—,
proceeds. 6. \$6.40, discount; \$793.60, proceeds. 7. \$16.32,
discount; \$1003.68, proceeds. 8. \$3.36, discount; \$556.64, pro-
ceeds. 9. \$10.80, discount; \$1189.20, proceeds. 10. \$17.918+,
discount; \$1611.015+, proceeds.

Page 187.—1. \$250. 2. Answers differ. 3. 4 yd. 1 ft. 10 in.
4. 9 bu. 2 pk. 5 qt. 5. 1685 $\frac{2}{3}$ revolutions. 6. 95¢. 7. 10 children.
8. $\frac{2}{3}$. 9. 583 $\frac{1}{2}$ bu. 10. 4 $\frac{1}{2}$ da. 11. \$26,250. 12. 40¢; 25¢.

Page 188.—13. \$36. 14. 105 A. 145 sq. rd. 15. 75 $\frac{1}{2}$ A.
16. \$7.87, amount of bill. 17. 5 da. 5 hr. 33 min. 20 sec. 18. 453 $\frac{1}{2}$ mi.
19. 2160 ft. 20. \$37.453+. 21. \$161.906+. 22. 5 lb. 2 oz. 23. 48
suits. 24. \$40. 25. 42 yd.; \$29.40. 26. 64 rd. long.

Page 189.—27. \$582.40. 28. 210; 280. 29. \$62,500. 30. \$633; \$844. 31. \$413. 32. \$55.20. 33. 2880 pills. 34. \$29. 35. \$5.04. 36. \$50; \$60. 37. \$39.20. 38. $\frac{3}{4}$. 39. $7\frac{1}{2}$ mi. 40. 13.4.

Page 190.—41. \$2420. 42. One hundred eighty-three hundred-thousandths. 43. 120 and 165. 44. \$9. 45. $\frac{1}{3}$ wk. 46. $7\frac{2}{3}$ tons. 47. \$957.60. 48. \$8.876+. 49. \$183.21. 50. $\frac{11}{15}$; $6\frac{2}{3}\%$. 51. 488. 52. 92,000 bricks. 53. $1\frac{1}{2}$ min. 54. \$10844.75. 55. \$294.

Page 191.—56. 137 da. 57. 55 rd. 3 yd. 2 ft. 3 in. 58. 11 A. $85\frac{7}{8}$ sq. rd. 59. $4\frac{1}{2}$ yd., $\frac{3}{4}$ yd., and $\frac{7}{12}$ yd.; sum, $5\frac{1}{2}$ yd. 60. 40%. 61. \$3650. 62. \$5.164+. 63. 20 rd. 65. \$2350. 66. 24 yd.; 27 yd. 67. \$502.22. 68. \$925.714+. 69. \$272.112+.

Page 192.—70. \$249.04. 71. 11.592 qt. 72. \$5.88. 73. 8.7. 74. \$6. 75. $2\frac{3}{4}$ cd. 76. $16\frac{1}{2}\%$. 77. \$92.436+. 78. \$47.58. 79. .0375; $\frac{3}{80}$. 80. $42\frac{3}{4}$ yd. 81. 3.9. 82. \$354.09. 83. 5 rd. 4 yd. 6 in. 84. \$54.60.

Page 193.—85. $\frac{2}{3}$ rd. 86. \$94.60. 87. $\frac{47}{185}$. 88. \$19.135; \$241.635. 89. $33\frac{1}{2}\%$. 90. \$108. 91. \$102.50. 92. 3 da. 3 hr. 12 min. 93. $6\frac{1}{2}\%$. 94. \$11.34. 95. $\frac{7}{8}$. 96. $\frac{1}{85}$ mi. 97. \$120.96. 98. \$367.449+. 99. 10 A. 100. \$4.75. 101. $53\frac{1}{2}\%$.

Page 194.—102. \$71.05. 103. $\frac{3}{8}\%$. 104. \$130.41. 105. \$32.062+. 106. \$1900. 107. 20%. 108. (a) $\frac{48}{800}$; (b) $\frac{1}{2}$; (c) $\frac{2}{800}$. 109. $\frac{7}{8}$. 110. 351 lb., copper; 99 lb., tin. 111. \$5420.80. 112. \$16,500. 113. \$151.20. 114. 110%. 115. \$22,500.

Page 195.—116. \$92.302. 117. 11 rd. 3 yd. 118. \$1.75. 119. $\frac{11}{100}$ cwt. 120. $\frac{1}{3}$ of 11. 121. \$1321.875. 122. $98\frac{3}{4}$ bu. 123. $\frac{7}{800}$; .00875. 124. \$151.20. 125. \$15.792. 126. \$52.00. 127. $6\frac{1}{2}$ da. 128. \$60. 129. \$19.85. 130. 292 lb.

Page 196.—131. $\frac{1}{2}$ bbl. 132. 1250. 133. 6 cd. 134. \$31.50. 135. \$69.30. 136. $2\frac{1}{2}\%$. 137. 1680 gal. 138. 80 apples. 139. 20%. 140. 7 A. 141. \$6000. 142. 5%. 143. \$344.75. 144. 4.03 P.M.

Page 197.—145. 88 ft. 146. $2\frac{1}{2}\%$. 147. 42. 148. \$159.96. 149. 4 bu. 3.44 pk. 150. \$16.50, first; \$22.50, second; \$15, third. 151. \$59.85. 152. \$522.05. 153. \$10.248+. 154. \$4,500. 155. $976\frac{1}{2}$ sq. ft. 156. \$723.35.

Page 198.—157. \$335.616+. 158. 2688 gal. 159. Sept. 13, 1900, date of maturity; 65 da., term of discount; \$3.25, discount; \$296.75, proceeds. 160. 269.1 cu. ft. 161. \$12.937+. 162. 9 hr. 163. \$8.84. 164. 8 mi. 165. $121\frac{1}{2}$ sq. ft. 166. \$4.111+. 167. 68.26 A. 168. 349.888+ bu.

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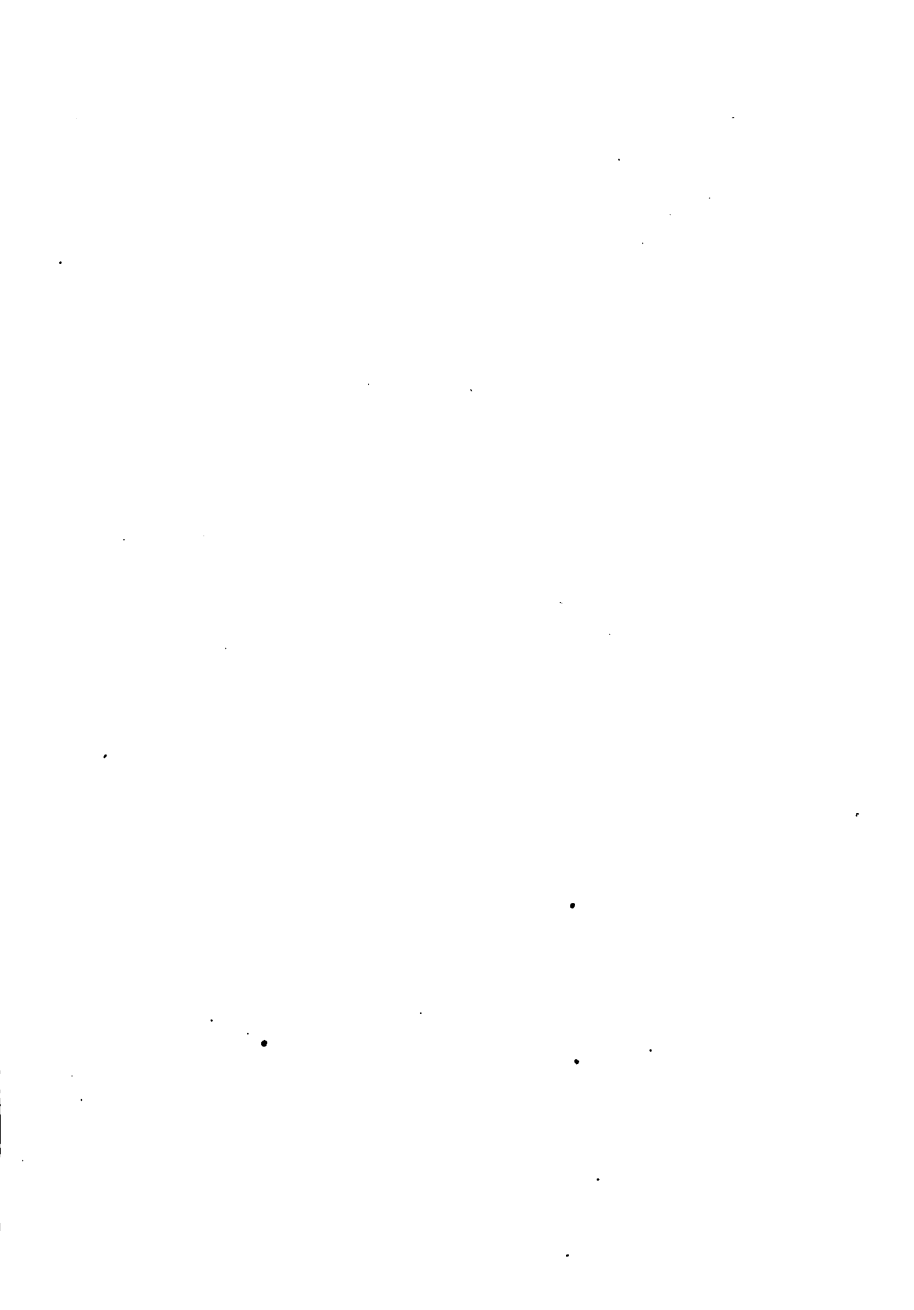
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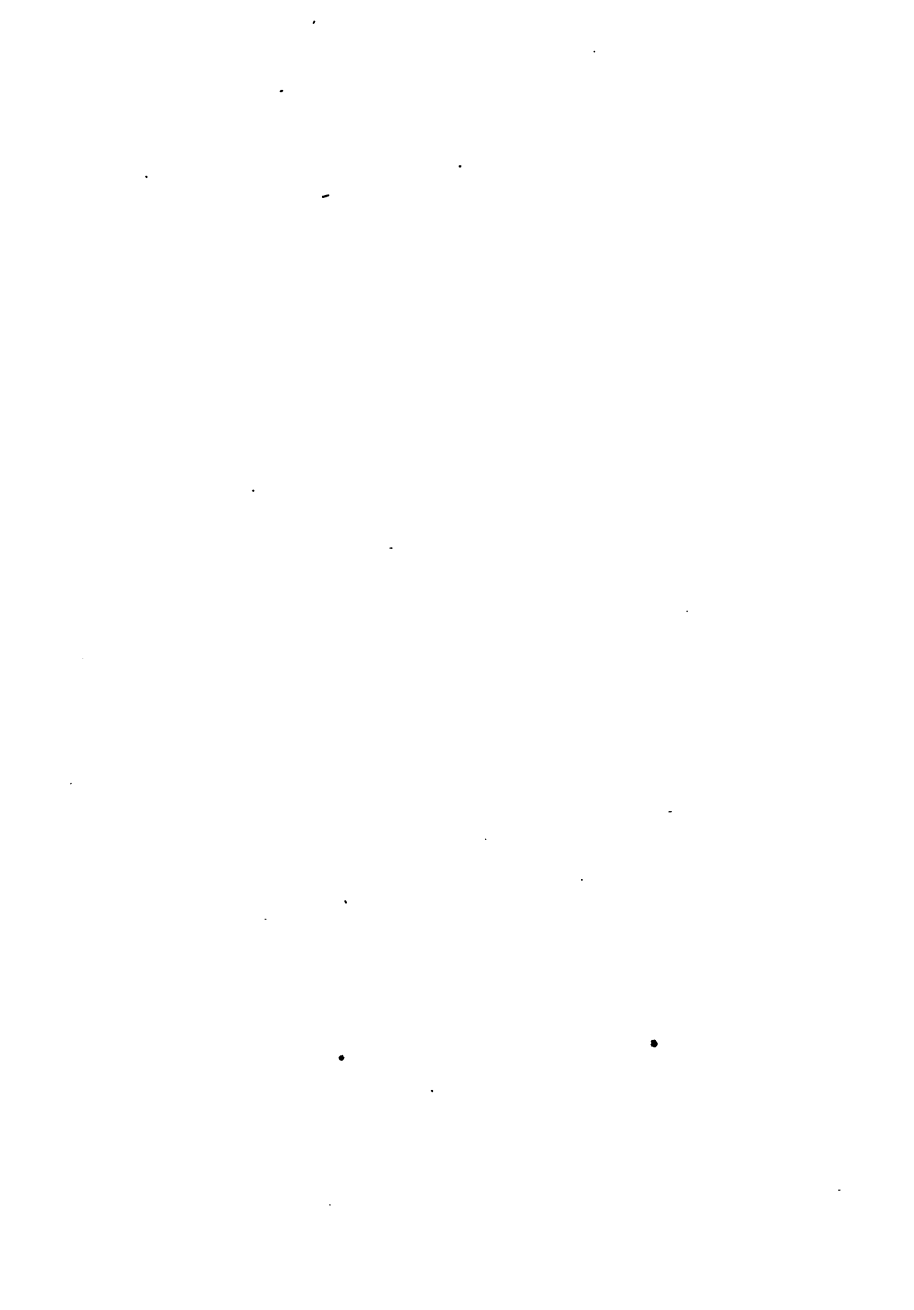
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